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Symmetric and Permutation Generating Sets of S_{28k+r} and A_{28k+r} of Degree $28k+r$ Using $PSL(2,27)$

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الملخص: هدف بحثنا الى استخدام زمرة $PSL(2,27)$ لتوليد زمرة التماثل S_{28k+r} و الزمرة A_{28k+r}. و توصلنا الا مجموعة من الزمر المتماثلة من S_{28k+r} و A_{28k+r} برتبة $28k+r$ و ذلك باستخدام المجموعة الخطية الإسقاطية $PSL(2,27)$ و حصلنا على $2k+r$ في S_{28k+r} لكل الاعداد الصحيحة عندما تكون $k \geq 1$. و لقد توصلنا أيضا أن S_{28k+r} و A_{28k+r} يمكن أن يتم إنشاؤها بشكل متناظر باستخدام بعض مجموعات توليد متماثلة.

الكلمات المفتاحية: زمرة التماثل، الزمرة الخطية $PSL(2,27)$.

Abstract: In this paper, we aimed to use $PSL(2,27)$ to generate the symmetric group S_{28k+r} and the alternating group A_{28k+r}. we have given symmetric and permutational generating sets of S_{28k+r} and A_{28k+r} of degree $28k+r$ using the projective linear group $PSL(2,27)$ and an element of order $2k+r$ in S_{28k+r} and A_{28k+r} for all integers $k \geq 1$. We also have shown that S_{28k+r} and A_{28k+r} can be symmetrically generated using some symmetric generating sets.

Keywords: the symmetric group, $PSL(2,27)$.

1.INTRODUCTION

The projective special linear group $PSL(2,27)$ is group of non-singular 2×2 matric over F_{27}. The $PSL(2,27)$ of order 9828 is one of the well know simple groups. It contains 16 conjugacy classes. It also contains four maximal subgroups of orders 351, 28, 26 and 12.

$PSL(2,27)$ can be generated using two permutations of orders 13 and 3 as follows;

$PSL(2,27) = \langle (3, 27, 25, 23, 21, 19, 17, 16, 14, 12, 10, 8, 6)(4,15, 13, 11, 9,7,5, 28, 26, 24, 22, 20, 18),(1, 2, 4)(5, 8, 24)(6, 21,10)(7, 16, 15)(9, 25, 28)(11, 13, 14)(12, 27,23)(17, 26, 18)(19,20, 22) \rangle.$

$PSL(2,27)$ can be finitely presented as follows [1,2]

$$PSL(2,27) = \langle x, y, t : x^3 = y^{13} = t^2 = (yt)^2 = (xt)^3 = 1, y^{-3}xy^3 = xy^{-1}xy = y^{-1}xyx \rangle.$$

In 1998, Al-Amri,[3,4], has shown A_{kn+1} and S_{kn+1} can be generated symmetric generating and symmetrically generating using S_n and an element of order *k*.In 1995, Al-Amri and Hammas [1], have shown that A_{kn+1} and S_{kn+1} can be generating using S_n and an element of order $k+1$ for all integers $n \geq 2$ and $k \geq 2$. Also, they have shown that A_{kn+1} and S_{kn+1} can be generated symmetrically using n elements each of order $k+1$. Al-Amri, Al-Shehri, Ashour and Al-Muhaimeed,([5-9])have studied different types of symmetric and permutational generating set of various groups using different projentors.

In this paper, we will show that S_{28k+r} and A_{28k+r} can be generated using the $PSL(2,27)$.

In this paper we generate the symmetric group S_{28k+r} and the alternating group A_{28k+r} using the projective special linear group $PSL(2,27)$.

We have generated the symmetric group S_{28k+r} and the alternating group A_{28k+r} using the simple group $PSL(2,27)$. We will introduce some definitions and known results for areas of group theory to be used in this paper. Also, many new results have been found to get large groups from small ones. In 2009, Al-Shehri,[4,10], has used the Mathieu groups M_9, M_{10}, M_{12} to get S_{kn+1} and A_{kn+1} .Also,Shafee,[11],has used the wreath product of group $PSL(2,13)\ wr\ PSL(2,11)$ by some other groups . Samman,[14], has used the projective special linear group $PSL(2,19)$ to get S_{20k+r} and A_{20k+r}

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2. PRELIMINARY RESULTS

Definition 2.1. [14] The general linear group $GL_n(q)$ consists of all the $n \times n$ matrices that have non-zero determinant over the field F_q with *q*-elements. The special linear group $SL_n(q)$ is the subgroup of $GL_n(q)$ which consists of all matrices of determinant one. The projective general linear group $PGL_n(q)$ and projective special linear group $PSL_n(q)$ are the groups obtained from $GL_n(q)$ and $SL_n(q)$. The projective special linear group $PSL_n(q)$ is also denoted by $L_n(q)$. The orders of these groups are:

$|GL_n(q)| = (q-1)N$, $|SL_n(q)| = |PGL_n(q)| = N$,

$|PSL_n(q)| = |L_n(q)| = \frac{N}{d}$,

where $N = q^{\frac{1}{2}n(n-1)}(q^n-1)(q^{n-1}-1)\dots(q^2-1)$

and $d = (q-1, n)$.

Definition 2.2.[9] A group G is said to be simple if G has no proper normal subgroup ; that is , G has no normal subgroups except {id} and G .

Definition 2.3.[13] If X is a nonempty set, the symmetric group on X , denoted by S_X , is the group whose element are the permutations of X and whose binary operation is composition of functions . Of particular interest is the special case when X is finite . If $X = \{1,2,3,\dots,n\}$, we write S_n instead of S_X , and we call S_n the symmetric group of degree n, or the symmetric group on n letters , of order n! .

Definition 2.4.[11] If X is a nonempty set . A subgroup G of the symmetric group S_X is called a permutation group on X . The degree of the permutation group is the cardinality of X .

Definition 2.5.[12] Two elements a and b are said to be conjugate in G if there is some $g \in G$ such that $b = g^{-1}a\,g$.

Theorem 2.1. [2] Let $1 \le a \ne b < n$ be any integer . Let n be an odd integer and let G be the group generated by the n-cycle $(1, 2, \dots, n)$ and 3- cycle (n, a, b). If the hcf(n,a,b)=1, then G = A_n .

Theorem 2.2. [2] Let G be the group generated by the *n*-cycle $(1, 2, \dots, n)$ and the involution (*n*, *a*)(*i* , *j*) for any *i* and *j*. Let $n \ge 9$ be an odd integer then $G \cong A_n$.

Theorem 2.3. [2] Let $1 < i \ne j < n$. Let $n \ge 8$ be an even integer. Let G be the group generated by the n-cycle $(1, 2, \dots, n)$ and the involution (n ,1)(i, j). If $(n,1)(i,j) \ne (n,1)(\frac{n}{2}, \frac{n}{2}+1)$ then $G \cong S_n$.

3. Generating the Symmetric Groups S_{28k+r} and the Alternating Groups A_{28k+r} Using $PSL(2,27)$

Theorem 3.1. $PSL(2,27)$ can be generated using two elements, the first is of order 14 and the second is of order 13.

Proof: Let $H = \langle \alpha, \beta \rangle$,where

α = (1, 26, 28, 13, 15, 2, 10, 5, 9, 17, 23, 7, 11, 6)(3, 20, 12, 18, 4, 16, 27, 19, 8, 21, 14, 25, 24, 22),
which is the product of two cycles each of order 14 and
β =(1, 2, 23, 25, 16, 24, 5, 6, 7, 13, 21, 12, 14)(3, 22, 10, 20, 18, 4, 27, 15, 8, 19, 26, 11, 17),
which is the product of two cycles each of order 13. We claim that H is $PSL(2,27)$. To show this, let

$\eta = \beta\alpha$ =(1, 10, 12, 25, 27, 2, 7, 15, 21, 18, 16, 22, 5)(4, 19, 28, 13, 14, 26, 6, 11, 23, 24, 9, 17, 20),

which is the product of two cycles each of order 13. Let

$\eta_1 = \eta^4$ =(1, 27, 21, 5, 25, 15, 22, 12, 7, 16, 10, 2, 18)(4, 14, 23, 20, 13, 11, 17, 28, 6, 9, 19, 26, 24),

which is the product of two cycles each of order 13. Conjugating η by $\alpha^{-4}\beta^6$ we get

η_2 = (1, 17, 2, 4, 9, 8, 22, 3, 6, 13, 27, 26, 18)(5, 21, 15, 19, 25, 23, 7, 28, 12, 10, 16, 20, 14).

Hence, we get the element

$x = \eta_1\eta_2^3$ =(3, 27, 25, 23, 21, 19, 17, 16, 14, 12, 10, 8, 6)(4, 15, 13, 11, 9, 7, 5, 28, 26, 24, 22, 20, 18) $\in H$, which is the first generator of $PSL(2,27)$.

Let :

$\mu_1 = (\alpha^9)^\beta$ =(1, 24, 22, 12, 27, 10, 19, 4, 5, 26, 14, 16, 15, 18)(2, 3, 8, 7, 9, 21, 17, 6, 28, 13, 20, 11, 25, 23),

which is the product of two cycles each of order 14,

$\mu_2 = (\alpha\beta)^2$ =(1, 7, 25, 9, 18, 26, 21, 11, 17, 5, 3, 27, 28)(2, 14, 15, 13, 12, 24, 6, 20, 16, 23, 8, 4, 10),

which is the product of two cycles each of order 13,

$\mu_3 = \beta^2$ =(1, 23, 16, 5, 7, 21, 14, 2, 25, 24, 6, 13, 12)(3, 10, 18, 27, 8, 26 , 17, 22, 20, 4, 15, 19, 11),

which is the product of two cycles each of order 13,

$\mu_4 = \alpha^2$ =(1, 28, 15, 10, 9, 23, 11)(2, 5, 17, 7, 6, 26, 13)(3, 12, 4, 27, 8, 14, 24)(16, 19, 21, 25, 22, 20, 18),

which is the product of four cycles each of order 7 and

$\mu_5 = (\beta^{-6})^{\alpha^3}$ =(1, 6, 10, 3, 24, 8, 16, 20, 22, 23, 13, 28, 9)(4, 15, 27, 26, 19, 11, 21, 18, 5, 12, 25, 17, 14),

which is the product of two cycles each of order 13. Hence, we get the element

$y = \prod_{i=1}^{5} \mu_i$ =(1, 2, 4)(5, 8, 24)(6, 21, 10)(7, 16, 15)(9, 25, 28)(11, 13, 14)(12, 27, 23)(17, 26, 18)(19,20, 22) $\in H$,

which is the second generator of $PSL(2,27)$.Therefore ,

$G = \langle x, y \rangle = PSL(2,27) \subseteq H$

On the other hand, let :

$\sigma = xy =$(1, 2, 4, 7, 8, 21, 20, 17, 15, 14, 27, 28, 18)(3, 23, 10, 24, 19, 26, 5, 9, 16, 11, 25, 12,6),

which is the product of two cycles each of order 13,

$\sigma_1 = (xy)^5 =$(1, 21, 27, 4, 17, 18, 8, 14, 2, 20, 28, 7, 15)(3, 26, 25, 10, 9, 6, 19, 11, 23, 5, 12, 24, 16),

which is the product of two cycles each of order 13,

$\sigma_2 = y^2 =$ (1, 4, 2)(5, 24, 8)(6, 10, 21)(7, 15, 16)(9, 28, 25)(11, 14, 13)(12, 23, 27)(17, 18, 26)(19, 22, 20),

which is the product of nine cycles each of order 3,

$\sigma_3 = y^{x^5} =$ (1, 2, 7)(3, 26, 28)(4, 9, 8)(5, 22, 6)(10, 11, 13)(12, 25, 21)(14, 27, 17)(15, 20, 23)(16, 18, 24),

which is the product of nine cycles each of order 3,

$\sigma_4 = x^y =$(1, 7, 14, 13, 25, 16, 8, 9, 18, 5, 19, 22, 17)(3, 23, 28, 12, 10, 20, 26, 15, 11, 27, 6, 24, 21),

which is the product of two cycles each of order 13, conjugating σ_4 by xy we get

$\sigma_5 =$ (1, 9, 26, 22, 15, 2, 8, 27, 13, 12, 11, 21, 16)(3, 19, 20, 23, 10, 18, 6, 24, 17, 5, 14, 25, 28),

which is the product of two cycles each of order 13 and

$\sigma_6 = (((x^y)^{xy} x)^{18} x^y)^5 =$(1, 22, 4, 18, 11, 17, 6)(2, 25, 14, 20, 13, 15, 9)(3, 5, 26, 27, 8, 28, 23)(7, 19, 10, 21, 12, 24, 16)

which is the product of four cycles each of order 7. Hence, we get the element

$\alpha = \sigma_1\sigma_2\sigma_3\sigma_6$ = (1, 26, 28, 13, 15, 2, 10, 5, 9, 17, 23, 7, 11, 6)(3, 20, 12, 18, 4, 16, 27, 19, 8, 21, 14, 25, 24, 22) $\in G = \langle x, y \rangle$.

Let :

$\delta_1 = (xy)^{10} =$(1, 27, 17, 8, 2, 28, 15, 21, 4, 18, 14, 20, 7)(3, 25, 9, 19, 23, 12, 16, 26, 10, 6, 11, 5, 24),

which is the product of two cycles each of order 13,

$\delta_2 = y^2 =$ (1, 4, 2)(5, 24, 8)(6, 10, 21)(7, 15, 16)(9, 28, 25)(11, 14, 13)(12, 23, 27)(17, 18, 26)(19, 22, 20),

which is the product of nine cycles each of order 3,

$\delta_3 = x^y = \sigma_4 = x^y =$ (1, 7, 14, 13, 25, 16, 8, 9, 18, 5, 19, 22, 17)(3, 23, 28, 12, 10, 20, 26, 15, 11, 27, 6, 24, 21),

which is the product of two cycles each of order 13 and

$\delta_4 = (x^{y^2} y^{xy} y^2 x^{17})^7 =$ (1, 24, 8, 28, 25, 4, 13, 7, 19, 3, 11, 18, 22)(2, 12, 16, 23, 5, 15, 27, 21, 17, 9, 6, 26, 10)

which is the product of two cycles each of order 13. we get the element

$\beta = \prod_{i=1}^{4} \delta_i$ =(1, 2, 23, 25, 16, 24, 5, 6, 7, 13, 21, 12, 14)(3, 22, 10, 20, 18, 4, 27, 15, 8, 19, 26, 11, 17) $\in G = \langle x, y \rangle$,

and therefore $H = \langle \alpha, \beta \rangle \subseteq G = \langle x, y \rangle$.

Hence, $H = \langle \alpha, \beta \rangle = G = \langle x, y \rangle = PSL(2,27)$.

Let $\theta : H \to \langle A, B \rangle$, where,

$A =$(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)(15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28) and

$B =$(1, 6, 11, 26, 20, 27, 8, 14, 12, 4, 24, 17, 25)(2, 13, 10, 15, 28, 7, 16, 18, 19, 21, 5, 23, 22) be the mapping which takes the point in the i^{th} position of α into i in A. It is not difficult to show that θ is isomorphism and therefore $PSL(2,27) \cong \langle A, B \rangle$. ◊

Theorem 3.2. S_{28k+r} and A_{28k+r} can be generated using $PSL(2,27)$ and an element of order $2k+r$ in S_{28k+r} and A_{28k+r} for all integers $k \geq 1$.

Proof: Let $G = \langle x, y, t \rangle$, where

x =(1, 2, 3, … , 14)(15, 16, 17, … , 28) … (28(k -1)+1, 28(k -1)+2, 28(k -1)+3, … , 28(k -1)+14)(28(k -1)+15, 28(k -1)+16, 28(k -1)+17, … , 28k),

which is the product of 2k cycles each of order 14,

y =(1, 6, 11, 26, 20, 27, 8, 14, 12, 4, 24, 17, 25)(2, 13, 10, 15, 28, 7, 16, 18, 19, 21, 5, 23, 22) … (28(k -1)+1,28(k -1)+6,28(k - 1)+ 11, … , 28(k -1)+25) (28(k -1)+2, 28(k -1)+13, 28(k -1)+10, … , 28(k -1)+22),

which is the product of 2k cycles each of order 13 and

t =(14, 28, … , 28(k -1)+14, 28k , 28k +1, …, 28k +r), which is a cycle of order 2$k+r$. Let $\sigma=tx$

σ=(1, 2, 3, 4, 5, 6,... , 28k , 28k +1, 28k +2,...,28k +(r -1), 28k +r)

which is a cycle of order 28k +r .We have the following two cases :

Case (1) : If r is an odd integer . For any $k \geq 1$, if r =1 then;

$\tau=\left[t^{x},t^{x^2}\right]^{-1}$ =(1, 2, 28k +1).

Since *hcf*(1, 2, 28k +1) =1, then by theorem 2.1 we get

$H=\langle\sigma,\tau\rangle\cong A_{28k+r}$.

While if r >1 then ;

$\delta_1=\left[t,t^{x}\right]$= (1, 14)(28$k$ +1, 28k +2).

Since σ is an even permutation, then by theorem 2.2

$H=\langle\sigma,\delta_1\rangle\cong A_{28k+r}$.

Case (2) : If r is an even integer . For any $k \geq 1$, then;

$\delta_2=\left[t,t^{x}\right]$= (1, 14)(28$k$ +1, 28k +2) .

Since σ is an odd permutation, then by theorem 2. 3

$H=\langle\sigma,\delta_2\rangle\cong S_{28k+r}$. ◊

Theorem 3.3. Let y and t be the permutations which have been described in theorem 3.2 . Let $G=\langle y,t\rangle$. Then $G\cong S_{26k+r}$ or A_{26k+r} for all integers.

Proof: Let $\sigma=ty$, it is not difficult to show that

σ=(1, 6, 11, 26, 20, 27, 8, 14, 7, 16, 18, 19, 21, 5, 23, 22, 2, 13, 10, 15, 28, 28(k -2)+12, 28(k -2)+4, 28(k -2)+24, 28(k -2)+17, 28(k -2)+25, 28(k -2)+1, 28(k -2)+6, 28(k -2)+11, 28(k -2)+26, 28(k -2)+20, 28(k -2)+27, 28(k -2)+8, 28(k -2)+14, 28(k -2)+7, 28(k -2)+16, 28(k -2)+18, 28(k -2)+19,28(k -2)+21, 28(k -2)+5, 28(k -2)+23,28(k -2)+22, 28(k -2)+2, 28(k -2)+13, 28(k -2)+10, 28(k -2)+15, 28(k -2)+28, 28(k -3)+12, 28(k -3)+4, 28(k -3)+24, 28(k -3)+17, 28(k -3)+25, 28(k -3)+1, 28(k -3)+6, 28(k -3)+11, 28(k -3)+26, 28(k -3)+20, 28(k -3)+27, 28(k -3)+8, 28(k -3)+14, 28(k -3)+7, 28(k -3)+16, 28(k -3)+18, 28(k -3)+19,28(k -3)+21, 28(k -3)+5, 28(k -3)+23,28(k -3)+22, 28(k -3)+2, 28(k -3)+13, 28(k -3)+10, 28(k -3)+15, 28(k -3)+28, ... , 28k , 28k +1, ... , 28k +r),

which is a cycle of order 26k +r . We have the following two cases :

Case (1) : If r is an odd integer . For any $k \geq 1$, if r =1, then;

$\tau_1 = \left[t, t^y\right]^y$ =(4, 12, 28k +1).

Let $H_1 \cong \langle \sigma, \tau_1 \rangle$. Let

$\theta : H_1 \to \langle (1,2, \ldots, 26k+r),\ (26k-2, 26k-3, 26k-4) \rangle$

be the mapping which has been described in theorem 3.1. Under this mapping and by theorem 2.1 we get $H_1 = \langle \sigma, \tau_1 \rangle \cong A_{26k+1}$.

While if r >1 then;

$\tau_2 = \left[t^y, t^{y^2}\right] =$ (4, 12)(28k +1, 28k +2).

Let $H_2 \cong \langle \sigma, \tau_2 \rangle$. Let

$\theta : H_2 \to \langle (1, 2, \ldots, 26k+r), (25k-1, 25k)(25k+(r-1), 25k+r) \rangle$ be the be the mapping which has been described in theorem 3.2. Under this mapping and by theorem 2.2 we get $H_2 = \langle \sigma, \tau_2 \rangle \cong A_{26k+r}$.

Case (2) : If r is an even integer . For any $k \geq 1$, then;

$\tau_3 = \left[t^y, t^{y^2}\right] =$ (12, 14)(28k +1, 28k +2) .

Let $H_3 \cong \langle \sigma, \tau_3 \rangle$. By theorem 2.3 it is not difficult to show that :

$H_3 = \langle \sigma, \tau_3 \rangle \cong S_{26k+r}$. ◊

4. Symmetric Generating Set of S_{28k+r} and A_{28k+r} :

Theorem 4.1. S_{28k+r} and A_{28k+r} can be symmetrically generated using the symmetric generating set $\Gamma = \{t_0, t_1, t_2, \ldots, t_{17}\}$, where $t_0 = t$ and $t_i = t^{x^i}$ for all integers $1 \leq i \leq 17$.

Proof: Let x been the element which has been described in theorem 3.2.

Let :

$t_0 = t$ =(14, 28, ... , 28(k -1)+14, 28k , 28k +1, ... , 28k + r),

$t_1 = t^{x^1}$ =(1, 15, ... , 28(k -1)+1, 28(k -1)+15, 28k +1, ... , 28k + r),

$t_2 = t^{x^2} =$ (2, 16, ... , 28(k -1)+2, 28(k -1)+16, 28k +1, ... , 28k + r),

.

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and

$t_{17} = t^{x^{17}} =$ (17, 31 ,... ,28(k -1)+3, 28(k -1)+17, 28k +1, ... , 28k + r).

Let $H = \langle \Gamma \rangle$. We claim that $H \cong S_{28k+r}$ or A_{28k+r} depending on whether r is an odd or an even integer respectively. To show this, consider the element;

$$\alpha = \prod_{i=1}^{17} t^{x^i}.$$

It is not difficult to show that,

$\alpha =$(1, 15, ... , 28(k -1)+1, 28(k -1)+15, 2, 16, 28(k -1)+2, 28(k -1)+ 16, 3, 17, ... , 28(k -1)+3, 28(k -1)+17, 4, 18, ... , 28(k -1)+4, 28(k -1)+18, 5, 19, ... , 28(k -1)+5, 28(k -1)+19, 6, 20, ... , 28(k -1)+6, 28(k -1)+20, 7, 21, ... , 28(k -1)+7, 28(k -1)+21, 8, 22, ... , 28(k -1)+8, 28(k -1)+22, 9, 23, ... , 28(k -1)+9, 28(k -1)+23, 10, 24, ... , 28(k -1)+10, 28(k -1)+24, 11, 25, ... , 28(k -1)+12, 28(k -1)+25, 12, 26, ... , 28(k -1)+12, 28(k -1)+26, ... , 13, 27, ... , 28(k -1)+13, 28(k -1)+27, 14, 28, 28(k -1)+14, 28k , 28k +1, ... , 28k + r),

which is a cycle of order 28k + r . Now, if r =1, then;

$\tau = [t_1, t_2]^{-1}$ = (1, 2, 28k +1) .

Since *hcf*(1, 2, 28k +1) = 1, then by theorem 2.1. we get

$$H = \langle \sigma, \tau \rangle \cong A_{28k+r}.$$

While, if r >1 is any integer, then;

$\delta = [t_1, t_2]$ = (1, 2)(28k +1, 28k +2) .

Hence, If r is an even integer, then; by theorem 2. 3 we get

$$H = \langle \sigma, \delta \rangle \cong S_{28k+r}.$$

While if r is an odd integer, then; by theorem 2.2 we get

$$H = \langle \sigma, \delta \rangle \cong A_{28k+r}. \Diamond$$

Theorem 4.2. Let $\Gamma = \{t_0, t_1, t_2, ..., t_{13}\}$ be the symmetric generating set of the groups S_{28k+r} and A_{28k+r} which have been described in the previous theorem . If we remove m -elements of the set Γ for all $1 \le m \le 12$ then the resulting set generates $S_{(28-2m)k+r}$ and $A_{(28-2m)k+r}$. If we remove 13 elements of the set Γ then the resulting set generates C_{2k+r}, depending on whether r is an odd or an even integer respectively.

Proof: : Let $\Gamma=\{t_0,t_1,t_2,\ldots,t_{13}\}$. Let $\Gamma_1=\{t_1\}$. It is clear that $\langle\Gamma_1\rangle\cong C_{2k+r}$, which is a cycle of order $2k+r$. Let $\Gamma_2=\{t_1,t_2\}$. Let $H=\langle\Gamma_2\rangle$. We claim that $H\cong S_{4k+r}$ or A_{4k+r}. To show this, let $\alpha=t_1t_2^{-1}t_1$. It is not difficult to show that,

$\alpha=$(1, 28(k -1)+1, 28k +1, 28k +2, ... , 28k + r , 15, ... , 28(k -1)+15, 28(k -1)+16, 28(k -1)+2, 16, 2),

which is a cycle of order $4k+r$. Now, if r =1, then;

$\tau=[t_1,t_2]^{-1}=$ (1, 2, 28k +1).

Let $H_1=\langle\alpha,\tau\rangle$. By theorem 2.1, It is not difficult to show that $H_1\cong A_{4k+r}$. Since t_1 and t_2 are even permutation and since H acts on $4k+r$ points, then $H\cong H_1\cong A_{4k+r}$.

While, if r >1 is any integer, then;

$\delta=[t_1,t_2]=$ (1, 2)(28k +1, 28k +2).

Let $H_2=\langle\alpha,\delta\rangle$. Then, If r is an even integer, then by theorem 2.3, $H_2\cong S_{4k+r}$. While if r is an odd integer, then by theorem 2.2, $H_2\cong A_{4k+r}$. Since H acts on $4k+r$ points, then $H\cong S_{4k+r}$ or A_{4k+r} depending on whether r is an odd or an even integer respectively. Therefore, The rest of the proof goes in the same way. ◊

5. SUMMARY:

In this paper we have given symmetric and permutational generating sets of S_{28k+r} and A_{28k+r} of degree $28k+r$ using the projective linear group $PSL(2,27)$ and an element of order $2k+r$ in S_{28k+r} and A_{28k+r} for all integers $k\geq 1$. We also have shown that S_{28k+r} and A_{28k+r} can be symmetrically generated using some symmetric generating sets.

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