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(Jensen's) Inequality for the frequent functions an application study

Oruba Ahmed Saleh Al Z'ubi

AL- Balqa Applied University || Jordan

Abstract: The researcher carried out a thorough and a deliberate study on (Jensen's) Inequality and its complements in the repeated linear relations, as well as he introduced a number of applications on the same, besides some improvements that have been introduced on some areas of the Inequality.

Keywords: Jensen's , Inequality, for the frequent functions.

Introduction:

The paul R.Beesack and Josip E Pecaric in 1985 defines a row for the frequent functions under certain conditions, then the norm for the frequent functions in order to make a generalization on the .mixed (Gessen for the frequent functions

The researcher studied (Jensen's) Inequality and its complements for the repeated relations, as in 1985, both researchers (Paul R. Beesak) and (Josip E. Pecaric) defined a linear of relations in order to make a generalization on (Jensen's) Inequality for the frequent functions. In 1986 both researchers made a study on some improvements that could be introduced on (Jensen's) Inequality for the repeated relations, and in 1991 they continued their research by providing some applications on (Jensen's) Inequality.

The study objective:

The aim of this study is to identify (Jensen's) Inequality for the repeated relations.

the importance of your study:

Recognition Through applications on (Jensen's) Inequality and its complements And improved.

your study improved this field:

The researcher studied (Jensen's) Inequality and its complements for the repeated relations.

Developing examples and establishing appropriate rules for them.

Methodology:

Applied descriptive approach

Application side:

(Jensen's) Inequality for the repeated relations:

a convex relation ϕLets (L) a linear that achieves the two properties (L1, L2) on the empty set (E), and let $\phi \in Lg$, so that on the non-empty interval $R \supset I$, if A is a repeated linear relation on L, so there shall be $g \in L$ shall be $A(g) \in I$, and the following Inequality shall be achieved:

$$\phi(A(g)) \leq A(\phi(g)) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2\ldots1)$$

Proof:

is a convex relation on the interval ϕLet $[\alpha, B) = I$, as

$(g) \text{ E } L\phi$ and $[\alpha,\beta]$

g, then: Includes $g \in I$

$=Aa(\alpha)\leq A(g)\leq A(\beta)=\beta$

is a convex relation on the interval 1, then for ϕ From the above, we conclude that $A(g)\in I$, and since m , so that:)each number x_0 from 1, there is a fix number = m (x_0

$$\phi(x)\geq\phi(x_0) \div m\ (x-x_0) \quad \in I \ X$$

If we put g=, $A\,x_0 = (g)x$, we shall get:

$$\phi(g)\geq\phi(A(g)) \div m(g-A\,(g))$$

By applying A on this Inequality, we find that:

$$A(\phi(g))\geq\phi(A(g))+ m(A(g)-A(g))$$

From that also:

$$A(\phi(g))\geq\phi(A(g))$$

And this is the QED.

In complements for (Jensen's) Inequality, we can prove the following theories, that include Inequalities (g) $\leq$ F (A (g), by a proper and definite selection of the relation F.(ϕ from the figure A

Let L a pattern that realizes the two properties L1, L2, on the non-empty set E, and let A a repeated linear a convex relation on the interval :ϕrelation applied on M, and let

$(-\infty<m<M<\infty)\ I = [\ m\ ,\ M]$

Then, for each $g\in I$.so that $\phi(g)\in I$

................(2-2)

$$A(\phi(g))\leq[(M-A(g))\phi(m)+(A(g)-m)\phi(M)]/(M-m)$$

Proof:

Since ϕ(g)∈L , so:

m≤g(t)≤M,

By identifying the convex relation, we find:

ϕ(v)≤ $\frac{w-v}{w-u}$ ϕ(u)+ $\frac{v-u}{w-u}$ϕ(w) ∀ u ≤v≤w,u<w

If we put u = m, w = M, g = v, we get:

ϕ (g) ≤ $\frac{M-g(t)}{M-m}$ ϕ(m)+ $\frac{g(t)-M}{M-m}$ ϕ(M)

By applying A on both of the Inequality ends, we get:

ϕ(M)A(ϕ(g))≤$\frac{M-A(g)}{M-m}$ϕ(m)+ $\frac{A(g)-m}{M-m}$

And this is QED

Let L a pattern that realizes the two properties (L1, L2) on the non-empty set E, and let A a repeated linear

1= [m,M], a convex relation for the intervalϕrelation on L, and let

(x) ≥ 0, and the equality is realized mostly at the isolated points from 1, and let's ϕ-∞<m<M<∞) , as (

assume that one of the following conditions has been achieved:

(i) ϕ(X)>0 (i) for each x in the interval 1.

(ii) ϕ(X)>0 for each m<x<M

And realize one of the following cases: ϕ (m)=0,

ϕ'(M)≠0 or ϕ (M)=0 , ϕ'(m)≠0

(iii) ϕ(X)<0 for x in the interval 1.

(i) ϕ(x)<0 for each m<x<M, and realizes one of the following cases only:

ϕ(m)=0 or

ϕ (M)=0

(g) ∈L, so the Inequality shall be:ϕThen, for each g∈L , so that

A(ϕ(g))≤π ϕ(A(g))..................

(0,1) in both cases (iii), (iv), π Shall be right for some of π>l values in both cases (i) & (ii) and ∈

particularly, we can calculate the value of π in the Inequality (2,3) in the following way:

We put ،μ=$\frac{\phi(M)-\phi(m)}{M-m}$

If ، μ =0 Is:

Assuming that X=$\bar{x}$

is the only solution for the equation - (m<$\bar{x}$<M) ϕ'(X)=0, then:

π =ϕ(m)/ϕ($\bar{x}$)

But if it is μ ≠0 , and let it be: x=$\bar{x}$

is the only solution for the interval [m,M] of the equation

μ ϕ(x)-ϕ'(x)(ϕ(m)+ μ (x-m))=0

Then π=μ /ϕ '($\bar{x}$)

And also we have m<$\bar{x}$<M in cases (i) and (iii).

(b) Let L a line that realizes the two propertiesL1, L2 on the non-empty set E, and let A a frequent linear a concave relation on the interval :ϕrelation and documented on L, and let

(- b ∞<m<M<∞)[m,M]), as ϕ '(x) ≤0 and realizes equality mostly on the isolated points

From 1. Furthermore, it realizes one of the following cases: (i), (ii), (iii), (iv), then for each g∈L , provided (g) ∈ L realizes the cross Inequality for the Inequality (2 – 3), as for π – it shall be calculated in ϕthat

π <1 . the same aforementioned method in (a). However, in this case it shall be

when ϕ(x) ϕ<0 in the interval (m,M), and 0< π <1 when:

ϕ(X)>0 in the interval (m,M).

Proof:

C(M,ϕ(M), which has the equation ،(a)Let BC the line that extends between the two points B(m,ϕ(m),

, then (x), as π >0 ϕh(x)=y=ϕ(m)+ μ (x-m), taking into consideration the set of the convex relations π we conclude from the theory(1,2,13) mentioned in chapter one, that there is 0< π the only one that realizes the theory conditions, so that the straight line h(x) in touch with the curve π ϕ (x) at the point:

($\bar{x}$, π ϕ($\bar{x}$)), so we shall have:

H(x)≤ π ϕ(x),∀ x∈I.

As g∈I, so A(g) ∈I, so we may put: A(g)=x, so as to obtain:

h(A(g))≤ π ϕ(A(g))

Furthermore, the theory (2,1,3) reveals that:

$$A(\phi(g)) \leq \frac{\phi(M)-\phi(m)}{M-m}(A(g)-m)+\phi(m)=h(A(g))$$

Therefore:

A(ϕ(g))≤h(A(g))≤ π ϕ(A(g))

a convex for this interval, so it ϕ 1=ϕ (x) is a concave for the interval 1, the relation ϕ(b)As the relation realizes all hypothesis in branch ( ), therefore, the Inequality (2,3) can be applied on the concave relation 1(x) to get : ϕ

A(ϕ1(g))≤ π ϕ_1(A(g))

Then:

-A(ϕ(g))≤- π ϕ(A(g))

So we shall have:

A(ϕ(g))≤ π ϕ(A(g))

1(x),0, that is when ϕwhen (x) <0 in the interval (m,M), 0< π <Iϕ, that is when $_1$(x)>0ϕAs π >I when (x)<0 in the interval (n,M).ϕ

This is QED.

Let L a pattern that realizes both properties (L1, L2) on the non-empty set A,E, a linear frequent

′(x) is totally ϕ (x) L, a derivative relation on the interval [m,M]=I, as ϕdocumented relation on

g)∈L :ϕ(increasing on 1, then for each g∈L, so that

A(ϕ(g))≤ π ÷ϕ(A(g))..................

0< π <(M-m)(μ –ϕ′(m) and μ= $\frac{\phi(M)-\phi(m)}{M-m}$

Particularly, π in the Inequality (2,4) may be calculated in the following way:

If

x=$\bar{x}$ is the only solution for the equation ϕ $'(x) =$ μ, as $m < \bar{x} < M$, $then$:

. π=ϕ (m)-ϕ (x̅)+ μ (x̅-m)

(b) Let L a pattern that realizes both properties (L1, L2) on the non-empty set E,A, a linear frequent

′(x) is totally ϕ (x), a derivative relation on the interval [m,M]=I, as ϕdocumented relation on L, and let

g)∈L shall be :ϕ(decreasing on 1, then for each g∈L, so that

ϕ(A(g))≤ π+A(ϕ(g)).............

As 0< π <(M-m)(ϕ′(m)- μ), and u as it is in (a), in fact we can take π =ϕ(x̅)-ϕ(m)- μ (x̅-m) as x̅ as it is in (a).

Proof: (a)The same case in proving the theory (2,1,4) we shall take the line equation BC, whereas C(M,ϕ(M), and B(m,ϕ(m) , which is:

÷ μ (x-m) x∈I h(x)=ϕ(m), and the set of the concave relations

π+ ϕ(x), as we shall have 0< π a unit that realizes the conditions of this theory, as detailed in the theory (1,2,15) stated in the first chapter, so that the straight line h(x) in touch with the curve π +ϕ(x) at the point, (x̅,,π+ϕ(x̅) , so we shall have:

h(x)≤ π ÷ϕ(x) ; x∈I

As g∈I , it is required that A(g) ∈I, so if we put X=A(g), we get:

h(A(g))≤ π + ϕ (A(g))

Also, we have from the theory (2,1,3):

$A(\phi(g)) \leq h\,(A(g)$

Building on that:

$A(\phi(g)) \leq h\,(A(g)\,) \leq \pi + \phi(A(g)$

which realizes all of the $(x)=-\phi\ (x)\phi_1(b)$If we apply the Inequality (2,4) on the convex relation hypothesis in (a), we shall get:

$A(\phi_1(g)) \leq \pi + \phi_1\,(A(g)\,)$

This means that:

$-A(\phi\,(g)) \leq \pi - \phi\,(A(g)\,)$

Then:

$\phi(A(g)) \leq \pi + A\,(\phi(g)$

As:

$0 < \pi < (M-m)(\phi'_1(m) - \mu_1) = (M-m)(\mu - \phi'(m)$

As, in this case (case b) it shall be:

$$\mu_1 = \frac{\phi 1(M) - \phi 1(m)}{M-m} = \frac{\phi(m) - \phi(M)}{M-m} = -\mu$$

$\Phi(\bar{x})-m) = \pi = \Phi_1(m) - \Phi_1(\bar{x}) + \mu_1(\bar{x}-(m)\Phi-(\bar{x})\mu-m)$

And this is the QED.

Let(L) a pattern that realizes both properties (L1, L2) on the non-empty set E,A, and let (A)a linear frequent (x) a convex relation on the interval 1ϕon L, and let $\frac{ua-A(pg)}{u-A(P)} \in I$, a, and on the group E, and $0<A(P)<U$, $(U \in R)$, and Let's assume that $P \in L$ so that $P \geq 0$ $p\phi(g) \in L$, then there shall be: , and $pg \in L$

$$\Phi\ \frac{ua-A(pg)}{u-A(P)} \geq \frac{u\phi(a)-A(p\phi(g))}{u-A(P)} \quad \ldots\ldots\ldots\ldots\ldots$$

Proof: :

If we put the Inequality

$\frac{A(pg)}{A(P)}b=,\ q=-A(p)p=u$

We get:

$$\phi(\ \geq u\phi(a) - A(p)\phi\,\frac{A(pg)}{A(p)})]/(u-A(p))\frac{ua-A(p)\,[A(pg)/A(p)]}{u-A(P)}$$

Now we take the relation $A_1(g)=A(pg)/A(p)$, which shall realize the following properties:

(1) If:

$K,h \in L, \in R\propto, \beta$

Then:

$A_1(\propto K+\beta h)=\frac{A(p\propto k+p\beta\, h)}{A(p)}=\frac{(\propto A(pk)}{A(p)}+\frac{(\beta A(pg)}{A(p)}=\propto A1(K)+$ A1(h) β

(2) If:

$>0a1(f)=\frac{A(pf)}{A(p)}$ F(t)>0 on the group E, then:

$\frac{A(p)}{A(p)}A_1(1)==1$

From this we conclude that A_1 is a linear repeated and relation,

From (Jensen's) Inequality, we shall have:

$\Phi(A_1(g))\leq A_1(g))\leq A_1(\phi(g))$

From this, there shall be:

$\phi(\ \frac{A(pg)}{A(p)}\ \leq\ \frac{A(p\phi(g))}{A(p)}$

So It shall be:

$U\phi(a)-A(P)\phi(\ \frac{A(pg)}{A(p)}\ )\geq U\phi(a)-A(p\phi(g))$

And this is the QED.

Through applications on (Jensen's) Inequality and its complements in the theory (2,1,3), we have the following theory, which is considered as a generalization to a theory that is related to (Lupas).

Let L a pattern that realizes the two properties L1, L2, on the non-empty set :

$(-\infty\ <a<b<\ \infty\ )\ E[\ a,b\ ]$

a convex relation on the group E, ϕAnd let A - a linear frequent documented on L, and assuming that $e_1\in L$, as $x(x)=x,\forall e_1$, then we have: ، $\in L\phi$and

$\Phi(A(e_1))\leq A(\phi)\leq[(b-A(e_1))\phi(a)+(A(e_1)-a)\phi(b)]/(b-a)$........

Proof:

$e_1\in L$ ، $\phi\ (x)=\phi(e_1)\in L$, then by using Jensen's Inequality with:

$g(x)=e_1(x)=x$, we get:

$\Phi(A(e_1))\leq A(\phi(e_1))=A(\phi(x)$

, so we may apply the theory (2,1,3) including in it $a\leq A(e_1)\leq$, then $a\leq e_1<b$, and b $(x)=\phi(e_1)$ ϕSince $M=a$ $M=b$ / $g(x)=e_1(x)=x$, so as to get:

$A(\phi(x))\leq b-A(e_1))\phi(a)+(A(e_1)-a)\phi(b)]/(b-a)$

Then we shall have:

$\Phi(A(e_1))\leq A(\phi(x))\leq[(b-A(e_1))\phi(a)+(A(e_1)-a)\phi(b)]/(b-a)$

And this is QED

Let (L) a pattern that realizes both properties (L1, L2) on the non-empty set E, and (A) is a linear frequent $[0,\infty]$ and R 1: f a relation that realizes (x) a convex relation on the interval ϕdocumented on L, and let the following condition:

$\Phi(x)\leq f(x)\leq C\phi(Bx)$, $x\in I$........................

As B,C>0 fixed two numbers, then for each $g\in L$, as $g\geq 0$ on the group E, and

(Bg),f(Bg)$\in$ ϕ

$F(A(g))\leq C(A(f(Bg)))$.........................

Proof: Since $g\geq 0$, we can put A(g) instead of x in the Inequality (2,8), so as to get:

$\phi(A(g))\leq f(A(g))\leq C\phi(BA(g))$

Whereas:

$c\phi(BA(g))=C\phi(A(Bg))$

(Bg)$\in$L , then we shall have :ϕSince Bg$\in$ L and

$c(\phi(A(Bg)))\leq CA((\phi(Bg)))$

Also, the same shall be in the Inequality (2,8):

$c(A(\phi(Bg)))\leq C(A(f(Bg)))$

From that we get:

$f(A(g))\leq C\phi(BA(g))=C(\phi(A(Bg)))\leq C(A(\phi(Bg)))\leq C(A(f(Bg)))$

And this is QED:

g$\in$, so then is a convex relation for the interval for each L ω The interval 1 from R, and (x)=xϕ(x) that:

A(g)>0 and $g^2,\phi(g)$ ، (g$\in$Lω , we shall have:

$A(\phi(g))\leq\phi(A(g))\leq A(g\phi(g)/A(g)<\phi(A(g^2)/A(g))$........

(g), and the relation ϕ – a convex relation on I, then Jenssen Inequality shall give us:ϕSince g$\in$ L ,

$A((\omega(g))\geq(\omega(A(g))$

That is:

$A(g\phi(g)\geq A(g)\phi(A(g))$

Or by using the relation:

$A_1(f)=A(fg)/A(g)$, whereas g,fg,$\in$L and A(g)>0

Which is a linear frequent and documented in Jenssen Inequality, as:

ϕ and g,g^2,ω (g)$\in$L is a convex relation, we get:

$-A(\phi(g))\geq-\phi(A_1(g))$

From that :

$\leq\frac{A(g\phi(g))}{A(g)}\phi\frac{A(g2)}{A(g)}$)

Therefore:

$\leq\frac{A(g\phi(g))}{A(g)}\leq A(\phi(g))\leq\phi(A(g))\phi\frac{A(g2)}{A(g)}$)

And this is the QED.

Theory (2, 2, 4):

Exp($\frac{A(g\ log\ g)}{A(g)}$)≥A(g)≥exp(A(log g))......................

Proof: Since ϕ(x)=log x is a concave relation and linked with each of x>0, and since

ϕ($\frac{1}{X}$)=-log x is a convex relation , then xϕ(x)=xlogx is a convex relation for each of x>0.

Moreover, since g,g log g ∈L ,g>0, then by using Jensen's Inequality with the convex relation ϕ($\frac{1}{X}$), secondly, we shall get:

A(g log g) ≥A (g) log A (g)

A(log g)≤log A(g)

And since A(g)>0, then:

≥logA(g)≥A(log g)$\frac{A(g\ log\ g)}{A(g)}$

Thus:

)≥A(g)≥exp (A(log g))$\frac{A(g\ log\ g)}{A(g)}$exp(

And this is the QED:

The theory (2 – 2 – 6):

, and f^s> $\frac{s-r}{s+r}$ ∈L, f^r Let f≥0 on the group E, as log f∈L, $\frac{1}{1\ \div\ fs}$

This means:

0≤ r ≤ s ≤ ∞ , then:

1/(1+(A(f^r))$\frac{s}{r}$)r>0..........≤A(1+f^s));

f>1, then:

A(1/1+f^S))≥1/[1+exp(sA(log f))]; r=0............

, and let Initially, we take r>0

ϕ(x)=(x o ω^{-1})(x) ، x (X) = $\frac{1}{1\ +\ xs}$ = ، (x)=x^r ω

x>0 , we notice that:

$^{-2}\frac{s}{r}$Φ''(x)=(1+x$\frac{s}{r}$)^2($\frac{-s}{r}$($\frac{s}{r}$-1)x)+$\frac{s}{r}$2(1+$x^{-1}$$\frac{s}{r}x\frac{s}{r}$)($\frac{s}{r}x\frac{s}{r}$-1)/(1+x$\frac{s}{r}$)^4

=$\frac{s}{r}$ (1+ x$\frac{s}{r}$) x$\frac{s}{r}$ $^{-2}$((1- x$\frac{s}{r}$) (1+X x$\frac{s}{r}$) +2- X$\frac{s}{r}$X$\frac{s}{r}$)/(1+X $\frac{s}{r}$)^4

So as to have:

ϕ″(X)>0, it shall be:

x$\frac{2S}{r}$+1x$\frac{s}{r}$( ) $\frac{s}{r}$>X$\frac{s}{r}$ - 1, that is: $\frac{s-R}{S+R}$>x$\frac{s}{r}$

Therefore, ϕ(X) is totally convex if R<S, $\frac{s-r}{s+r}$> X $\frac{s}{r}$, and since $f^r \in L$

$L\in\Phi(f^r)=\frac{1}{1+f\,S}$

$A(\phi(f^r))\geq\phi(A(f^r))$

So,

$A(\frac{1}{1+f\,S})\geq\frac{1}{1+(A(f\,r))\frac{s}{r}}$

, we also find: In the case r = 0, we take (x)ω = log x, as x>0, we find that $\phi(x)=\frac{1}{1+e5x}$ ϕ(x)=

$\Phi''(x)=[s^2e^{sx}(1+e^{sx})(e^{sx}-1)]/(1+e^{sx})^4$

we must have $e^{sx}-1>0$, from which sx>0, that is s>0, ″(x)>0ϕIn order to have

From this we conclude that ϕ(x) is a totally convex relation, and by using Jensen's Inequality with g = log f, we shall get:

$(\frac{1}{1+fs})\geq\frac{1}{1+\exp(s\,(A(\log f)))}$ A

And this is the QED.

Let f(x) a relation, so as $\phi(x)=\frac{1}{1+f(x)}$

to be a convex relation then for each relation of $g\in L$

shall be: (g)∈LϕAs

$\leq A(\frac{1}{1+f(g)})$ .... $\frac{1}{1+f(A(g))}$

A repeated linear function on L , and let it be $g_1\geq0$ for each i=1,2,3,.......,n, as:

$L\in{}^{P}g_i,g_i^{P}(\sum_{i=1}^{n}g\,i)$

And 0≠P∈R

If p>1

Then:

$(g_i^P)\leq(\sum_{i=1}^{n}$ $A\frac{1}{P}((\sum_{i=1}^{n}g\,i)^P)\,A\frac{1}{P}$

Proof: Let's take the function:

$a=(a1,....an)\ \frac{1}{r})a_i^r\Phi(a)=(\sum_{i=1}^{n}$

A positive concessive r <1

$\propto X+(1-\Phi(\propto)\ \frac{1}{r}y)=(\sum_{i=1}^{n}$ $(\propto x_i+(1-\propto)y_i)^r$)

$\geq(+(\sum_{i=1}^{n}$ $((1-\propto\frac{1}{r}x_i)^r)\propto\sum_{i=1}^{n}$ $)Y_i)^r)\frac{1}{r}$

$\propto\geq(+(1-\propto)(\sum_{i=1}^{n}$ $(y_i)^r)\frac{1}{r}\frac{1}{r}(x_i)^r)\sum_{i=1}^{n}$

= Φ (x)+(1-∝∝) ϕ(y)

A(ϕ(f))≤ϕ(A(f))

A($\sum_{i=1}^{n}$ f_i^r) $\frac{1}{r}$≤($\sum_{i=1}^{n}$ $A^r(f_i)$) $\frac{1}{r}$

convex, and if we put $g_i=f_i^r$, r=$\frac{1}{p}$ ϕWe shall get the reverse Inequality when

ϕ(f)=($\sum_{i=1}^{n}$ g_i)^{P,} and when 0<r<1 is p>1, then we shall have: ∈1, and $f_i=g_i^P$ Then

$^P(g_n^P)$]A$\frac{1}{p}$(g_2^P)+...g_i^P)+A$\frac{1}{p}$A($g_1+g_2+\ldots g_n$)^P≤[A$\frac{1}{p}$

Therefore:

$(g_1+g_2+\ldots g_n)^P \leq A\frac{1}{p} g_i^P)+A\frac{1}{p}(g_2^P)+\ldots A\frac{1}{p}(g_n^P)A\frac{1}{p}$

And when R<0, then R<0 ,then we get:

$(g_1+g_2+\ldots g_n)^P \geq A\frac{1}{p} g_i^P)+A\frac{1}{p}(g_2^P)+\ldots A\frac{1}{p}(g_n^P)A\frac{1}{p}$

,then we get: And when r≥1, then 0<p≤1

$(g_1+g_2+\ldots g_n)^P \geq A\frac{1}{p} g_i^P)+A\frac{1}{p}(g_2^P)+\ldots A\frac{1}{p}(g_n^P)A\frac{1}{p}$

And this is the QED.

Conclusion:

The researcher carried out a thorough and a deliberate study on (Jensen's) Inequality and its complements in the repeated linear relations, as well as he introduced a number of applications on the same, besides some improvements that have been introduced on some

areas of the Inequality.

Lets (L) a linear that achieves the two properties (L1, L2) on the empty set (E), and let ϕ a convex relation on the non-empty interval R ⊐I , if A is a repeated linear relation on L, so there shall be ∈Lg , so that ϕ g ∈ L shall be A(g) ∈ I, and the following Inequality shall be achieved.

From the above, we conclude that A (g)∈I, and since ϕ is a convex relation on the interval 1, then for each number x_0 from 1, there is a fix number m $=(x_0)$ =m, so In complements for (Jensen's) Inequality, we can prove the following theories, that include Inequalities from the figure A(ϕ (g) ≤ F (A (g), by a proper and definite selection of the relation F

Let L a pattern that realizes the two properties (L1, L2) on the non-empty set E, and let A a repeated linear relation on L, and let ϕ a convex relation for the interval 1= [m,M],

(-∞<m<M<∞) , as ϕ (x) ≥ 0, and the equality is realized mostly at the isolated points from 1, and let's assume that one of the following conditions has been achieved

Shall be right for some of π>I values in both cases (i) & (i) and ∈ π (0,1) in both cases (iii), (iv), particularly, we can calculate the value of π in the Inequality (2,3) in the following way

Let BC the line that extends between the two points B(m,ϕ(m), ، C(M,ϕ(M), which has the equation h(x)=y=ϕ(m)+ μ (x-m), taking into consideration the set of the convex relations π ϕ (x), as π >0 , then we conclude from the theories (1,2. 3) mentioned in the chapter one, that there is 0< π the only one that realizes the theory conditions

As the relation ϕ (x) is a concave for the interval 1, the relation ϕ 1=ϕ a convex for this interval, so it realizes all hypothesis in branch ( ), therefore, the Inequality (2,3) can be applied on the concave relation ϕ 1(x) to In order to have ϕ ″(x)>0 , we must have e^{sx}-1>0, from which sx>0, that is s>0,

From this we conclude that ϕ(x) is a totally convex relation, and by using Jensen's Inequality with g = log f, we shall

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ملخص الدراسة:

هدفت هذه الدراسة لدراسة متباينة (جسن) ومتمماتها في فضاء الاقترانات الخطية المتواترة الطبيعية وغير الطبيعية، وقدمنا تطبيقات عليها.

كما تمّ دراسة التحسينات لمتباينة جسن في الفضاء المتواتر مع وضع الأمثلة والقواعد الرئيسية التي تساعد في تطبيقها وتضمين محتواها، واتبعت الدراسة المنهج التطبيقي الذي يساعد ف تفسير وإيضاح النتائج.

كما تمّ تطبيق مجموعة من الأمثلة الرئيسة التابعة لمنهج متابينة (جسن) ومتمماتها .
