Journal of Natural Sciences,
Life and Applied Sciences
Issue (4), Volume 1
December 2017
ISSN: 2518-5780

المجلة العربية للعلوم و نشر الأبحاث
Arab Journal of Sciences & Research Publishing
AJSRP

Homothetic Motion in a Bianchi Type –V Model in Lyra Geometry

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Abstract: In this paper we study a homothetic vector field of a Bianchi type-V model based on Lyra geometry. The aim of this paper is to get the components of homothetic vector field in Lyra geometry for Bianchi type-V, compare between it and the case of Bianchi type-I. Using ordinary method and Computer program to get the components of the vector . The results are compatible in the two methods and get the condition that the results tends to the case of Bianchi type-I. The cases when a displacement vector is a function of t and when it is constant are considered. In both two cases we investigate the equation of state.

Keywords: Bianchi type-V; homothetic vector field; Lyra geometry; Christoffal symbols; Riemannian Geometry.

I) Introduction:

The theme of symmetries of space-time is nearly as old as the outset of the theory of General Relativity. Symmetries play an important role in the study of space –time, because of their interest from both geometric and physical viewpoints. Symmetries have been studied in the theory of General Relativity based on Riemannian geometry. In the theory of general relativity different kinds of symmetries like isometry, homothetic, conformal, Ricci collineation and matter collineation have been extensively studied.[1- 8]

Lyra geometry [9] has proposed a modification of Riemannian geometry by introducing a gauge function into the structure less manifold that bears a close resemblance to Weyl's geometry. Several authors have studied cosmological models based on Lyra's manifold with constant and time-dependent displacement vectors . The paper is organized as follows: In the next section we summarize some of the basic concepts of Lyra Geometry, which will be used though this work' Section 3 deals with the model and evaluating the homothetic vector field. In section 4 the Maple results are shown.

Problem statement and objectives:

Where studying some of the models of metric space times in the **Lyra** geometry it is difficult to get homothetic equations as well as to get solved. In this paper we calculate the equations in the ordinary method as well as using a computer program and compare the results to be able to use computer programs in difficult models.

Methods: In this paper we get homothetic equation by equation (2.4). We solve the partial differential equations by separate variables and use Maple 17 program for getting the homothetic vector field.

II) The version of model and homothetic field in Lyra's geometry

An n-dimensional Lyra manifold M is a generalization to the Riemannian manifold [10],[11]

For any point $p \in M$ we can define the coordinate system $\{x^{\mu}\}_{\mu=1}^{n}$. In addition to these coordinate there exist a gauge function $x^{o} = x^{o}(x^{\mu})_{\mu=1}^{n}$ which together with $\{x^{\mu}\}_{\mu=1}^{n}$ form a reference system transformation $(x^{o}, x^{\mu})_{\mu=1}^{n}$. In Lyra geometry the metric or the measure of length of displacement vector $\zeta^{\mu} = x^{o} dx^{\mu}$ between two points $p(x^{\mu})_{\mu=1}^{n}$ and $q(x^{\mu} + dx^{\mu})_{\mu=1}^{n}$ is given by absolute invariant under both gauge function and coordinate system written as:

$$ds^2 = g_{\mu\nu} x^{o}\, dx^{\mu} dx^{o} dx^{\nu}$$

Where $g_{\mu\nu}$ is a metric tensor as in Riemannian connection $\left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\}$, but also by a function ϕ_{μ}, which arises through gauge transformation and it is given by

$$\left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} = \frac{1}{2} g^{\alpha\rho}(g_{\rho\nu,\mu} + g_{\rho\mu,\nu} - g_{\mu\nu,\rho}) \qquad (2.1)$$

$$\Gamma^{\alpha}_{\mu\nu} = (x^{o})^{-1} \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} + \frac{1}{2}\left(\delta^{\alpha}_{\mu}\phi_{\nu} + \delta^{\alpha}_{\nu}\phi_{\mu} - g_{\mu\nu}\phi^{\alpha}\right) \qquad (2.2)$$

Where ϕ is called a displacement vector field and satisfies $\phi^{\alpha} = g^{\alpha\beta}\phi_{\beta}$, we consider ϕ to be a timelike vector where

$$\phi_{\mu} = (\beta(t), 0,0,0) \qquad (2.3)$$

Throughout the paper M will denote a 4-dimensional Lyra manifold with Lorentz metric g which is a generalization to the 4-dimensional Riemannian manifold [12],[13]

As in Riemannian geometry , a global vector field $\zeta = \zeta^{\mu}(t, x, y, z)_{\mu=1}^{4}$ on M is called homothetic vector field if the following condition

$$\mathcal{L}_{\eta} g_{\mu\nu} = g_{\rho\nu} \nabla_{\mu} \eta^{\rho} + g_{\mu\rho} \nabla_{\nu} \eta^{\rho} = 2\psi g_{\mu\nu} \qquad (2.4)$$

holds where ψ is constant (the homothetic constant)on M, $\mathcal{L}$ denote a Lie derivatives and ∇ is the covariant derivative such that:

$$\left. \begin{aligned} \nabla_{\mu}\zeta^{\rho} &= \frac{1}{x^{o}} \partial_{\mu}\zeta^{\nu} + \Gamma^{\nu}_{\mu\alpha}\zeta^{\alpha} \\ \nabla_{\mu}\zeta_{\rho} &= \frac{1}{x^{o}} \partial_{\mu}\zeta_{\rho} - \Gamma^{\alpha}_{\mu\nu}\zeta_{\alpha} \end{aligned} \right\} \qquad (2.5)$$

Where $\Gamma^{\alpha}_{\mu\nu}$ is a Lyra connection form given by (2.1) in equation (2.2). where ψ is a constant, then ξ is called homothetic vector field and equation (2.4) called homothetic equation. If $\psi = 0$, then (2.4) is called Killing equation and ξ is called a Killing vector field on M.

III)The model and homothetic vector field

Consider the space time metric Bianchi type –V of the form:

$$ds^2 = dt^2 - Adx^2 - e^{2mx}[B^2dy^2 + C^2dz^2]$$

Where A, B and C are functions of t alone , and m is constant.

Where $x^0 = t, x^1 = x, x^2 = y, x^3 = z$

The study of homothetic vector fields, $\zeta = \zeta^\mu(t,x,y,z)_{\mu=1}^4$, we choose the normal gauge $x^o = 1$, we get the connection form in Lyra geometry from (2.2) as:

$\Gamma_{00}^0 = \frac{\beta(t)}{2}$ $\Gamma_{11}^0 = A(t)\frac{dA(t)}{dt} + \frac{\beta(t)}{2}A^2(t)$ $\Gamma_{22}^0 = B(t)\frac{dB(t)}{dt} + \frac{\beta(t)}{2}e^{2mx}B^2(t)$

$\Gamma_{33}^0 = e^{2mx}C(t)\frac{dC(t)}{dt} + \frac{\beta(t)}{2}e^{2mx}C^2(t)$ $\Gamma_{01}^1 = \frac{1}{A(t)}\frac{dA(t)}{dt} + \frac{\beta(t)}{2}$ $\Gamma_{22}^1 = -me^{2mx}\frac{B^2(t)}{A^2(t)}$

$\Gamma_{33}^1 = -me^{2mx}\frac{C^2(t)}{A^2(t)}$ $\Gamma_{02}^2 = \frac{1}{B(t)}\frac{dB(t)}{dt} + \frac{\beta(t)}{2}$ $\Gamma_{12}^2 = \Gamma_{13}^3 = m$ $\Gamma_{03}^3 = \frac{1}{C(t)}\frac{dC(t)}{dt} + \frac{\beta(t)}{2}$

At $\beta(t) = 0$ we get the Christoffal symbol of 2nd kind in Riemannian geometry

Now we use equation (2.4) to deduce the following system equations in Lyra geometry :

$$\zeta_{,0}^0 + \frac{\beta}{2}\zeta^0 = \psi \quad (3.1)$$

$$\zeta_{,1}^0 - A^2\zeta_{,0}^1 = 0 \quad (3.2)$$

$$\zeta_{,2}^0 + -B^2e^{2mx}\zeta_{,0}^2 = 0 \quad (3.3)$$

$$\zeta_{,3}^0 - C^2e^{2mx}\zeta_{,0}^3 = 0 \quad (3.4)$$

$$\zeta_{,1}^1 + (\frac{\dot{A}}{A} + \frac{\beta}{2})\zeta^0 = \psi \quad (3.5)$$

$$B^2e^{2mx}\zeta_{,1}^2 + A^2\zeta_{,2}^1 = 0 \quad (3.6)$$

$$C^2e^{2mx}\zeta_{,1}^3 + A^2\zeta_{,3}^1 = 0 \quad (3.7)$$

$$\zeta_{,2}^2 + (\frac{\dot{B}}{B} + \frac{\beta}{2})\zeta^0 + m\zeta^1 = \psi \quad (3.8)$$

$$C^2\zeta_{,2}^3 + B^2\zeta_{;3}^2 = 0 \quad (3.9)$$

$$\zeta_{,3}^3 + (\frac{\dot{c}}{c} + \frac{\beta}{2})\zeta^0 + m\zeta^1 = \psi \quad (3.10)$$

In the case of $\beta = 0$ we get the homothetic system equation in Riemannian Geometry.

By solving equation (3.1) as a linear equation we get:

$$\zeta^0 = \left[\int \psi e^{\frac{1}{2}\int \beta(t)dt}\, dt + c_0\right] e^{\frac{1}{2}\int -\beta(t)dt}$$, c_0 is the constant of integration (3.11)

Use (3.11) into (3.2):

$$\frac{\partial\{\left[\int \psi e^{\frac{1}{2}\int \beta(t)dt}\, dt + C_0\right] e^{\frac{1}{2}\int -\beta(t)dt}\}}{\partial x} - A^2\frac{\partial \zeta^1}{\partial t} = 0$$

$$-A^2\partial\zeta^1 = 0;$$

$$\int \partial\zeta^1 = constant\ function\ with\ respect\ \ to\ t\ ;\ \zeta^1 = F_1(x,y,z),$$

By the same method use (3.11) into (3.3). (3.4) we get:

$$\zeta^2 = F_2(x,y,z),\ \ \zeta^3 = F_3(x,y,z)$$

Where $F_1(x,y,z), F_2(x,y,z), F_3(x,y,z)$ are arbitrary functions which are to be determined.

Differentiating equation (3.5) with respect to t :

$$\left(\frac{\dot{A}}{A} + \frac{\beta}{2}\right)\zeta^0_{,0} = 0;\ \int \left(\frac{\dot{A}}{A} + \frac{\beta}{2}\right) d\zeta^0 = constant = \text{a};\ \ \left(\frac{\dot{A}}{A} + \frac{\beta}{2}\right) = \frac{a}{\zeta^0}$$

using (3.11) we get:

$$\frac{1}{A(t)}\frac{dA(t)}{dt} + \frac{\beta(t)}{2} = \frac{a}{\zeta^0} = \frac{a}{\left[\int \psi e^{\frac{1}{2}\int \beta(t)dt} dt + C_0\right] e^{\frac{1}{2}\int -\beta(t)dt}} \qquad (3.12)$$

Use (3.12) back into (3.5)we get

$$\zeta^1_{,1} + a = \psi\ ;\ \zeta^1_{,1} = \psi - a\ ;\ \frac{d\zeta^1}{dx} = \psi - a;$$

$$\zeta^1 = (\psi - a)x + c_1 \qquad (3.13)$$

Differentiating equation (3.8)and (3.10) with respect to t we get:

$$\frac{1}{\mathrm{B(t)}}\frac{\mathrm{dB(t)}}{\mathrm{dt}} + \frac{\beta(\mathrm{t})}{2} = \frac{\mathrm{b}}{\zeta^0} = \frac{\mathrm{b}}{\left[\int \psi e^{\frac{1}{2}\int \beta(t)dt} dt + C_0\right] e^{\frac{1}{2}\int -\beta(t)dt}} \qquad (3.14)$$

$$\frac{1}{\mathrm{C(t)}}\frac{\mathrm{dC(t)}}{\mathrm{dt}} + \frac{\beta(\mathrm{t})}{2} = \frac{\mathrm{d}}{\zeta^0} = \frac{\mathrm{d}}{\left[\int \psi e^{\frac{1}{2}\int \beta(t)dt} dt + C_0\right] e^{\frac{1}{2}\int -\beta(t)dt}} \qquad (3.15)$$

from (3.14),(3.8) and (3.13):

$$\zeta^2_{,2} + \frac{\mathrm{b}}{\zeta^0}\zeta^0 + \mathrm{m}[(\psi - \mathrm{a})\mathrm{x} + \mathrm{c}_1] = \psi;$$

$$\zeta^2 = \{(\psi - \mathrm{b}) - \mathrm{m}[(\psi - \mathrm{a})\mathrm{x} + \mathrm{c}_1]\}\mathrm{y} + \mathrm{c}_2 \qquad (3.16)$$

Then ζ^2 is a function depends on y and x

from (3.15), (3.10) and (3.13):

$$\zeta^3 = \{(\psi - d) - m[(\psi - a)x + c_1]\}z + \mathrm{c}_2 \qquad (3.17)$$

ζ^3 is a function depends on z and x, Where c_1, c_2 and c_3 are constants of integrations .

Without loss of generality, we assume that $\mathrm{c}_0 = \mathrm{c}_1 = \mathrm{c}_2 = \mathrm{c}_3 = 0$ therefore from equations (3.11),(3.13) , (3.16) and (3.17) we obtain the following homothetic vector field

$$\zeta = \left[\int \psi e^{\frac{1}{2}\int \beta(t)dt}\, dt\right] e^{\frac{1}{2}\int -\beta(t)dt}\,\partial t + (\psi - a)x\,\partial x +$$

$$+\{(\psi - b) - m[(\psi - a)x]\}y\,\partial y + \{(\psi - d) - m[(\psi - a)x]\}z\,\partial z \qquad (3.18)$$

Now we discuss the case when the displacement vector is constant ($\beta = \text{constant}$).

From (3.6)

$$\zeta^0 = \left[\int \psi e^{\frac{1}{2}\int \beta dt}\, dt + c_0\right] e^{\frac{1}{2}\int -\beta dt} = \left[\frac{1}{\frac{1}{2}\beta}\int \frac{1}{2}\beta\psi e^{\frac{1}{2}\beta t}\, dt + c_0\right] e^{\frac{1}{2}\int -\beta dt}$$

$$\zeta^0 = \left(\frac{2\psi}{\beta} e^{\frac{1}{2}\beta t} + c_0\right) e^{-\frac{\beta t}{2}}$$

Put $c_0 = 0$ *then* $\zeta^0 = \frac{2\psi}{\beta}$

Then the homothetic vector field from (3.18)

$$\zeta = \frac{2\psi}{\beta}\partial t + (\psi - a)x\,\partial x + \{(\psi - b) - m[(\psi - a)x]\}y\,\partial y + \{(\psi - d) - m[(\psi - a)x]\}z\,\partial z$$

From (3.12)

$$\frac{1}{A(t)}\frac{dA(t)}{dt} + \frac{\beta}{2} = \frac{a}{\left(\frac{2\psi}{\beta} e^{\frac{1}{2}\beta t} + c_0\right) e^{-\frac{\beta t}{2}}}$$

$$\int \frac{dA(t)}{A(t)} + \int \frac{\beta}{2} dt = \int \frac{a e^{\frac{\beta t}{2}}}{\left(\frac{2\psi}{\beta} e^{\frac{1}{2}\beta t} + c_0\right)} dt + ln a_0$$

$$ln\frac{A(t)}{a_0} - ln\left(\frac{2\psi}{\beta} e^{\frac{1}{2}\beta t} + c_0\right)^{\frac{a}{\psi}} = \frac{\beta}{2} t$$

$$A(t) = a_0\left(\frac{2\psi}{\beta} e^{\frac{1}{2}\beta t} + c_0\right)^{\frac{a}{\psi}} e^{-\frac{\beta}{2} t}$$

By the same steps with equation (3.14) and (3.15) we get

$$B(t) = b_0\left(\frac{2\psi}{\beta} e^{\frac{1}{2}\beta t} + c_0\right)^{\frac{b}{\psi}} e^{-\frac{\beta}{2} t}$$

$$C(t) = k_0\left(\frac{2\psi}{\beta} e^{\frac{1}{2}\beta t} + c_0\right)^{\frac{d}{\psi}} e^{-\frac{\beta}{2} t}$$

a_0 , b_0, k_0 is the constant of integration

If a=b=d in (3.12), (3.13) and (3.14) and $a_0 = b_0 = k_0$, we get

$$A^2(t) = B^2(t) = C^2(t) = a_0\left(\frac{2\psi}{\beta} e^{\frac{1}{2}\beta t} + c_0\right)^{\frac{2a}{\psi}} e^{-\beta t} \qquad (3.19)$$

So in Lyra geometry if a displacement vector is constant, a Bianchi-type V space time with scalar factors are functions of t only is a Friedman, Robertson-Walker (FRW) Models in Cosmology with scalar factor behaves as (3.19) :[14-16]

To compare our results with that obtained in general relativity based on Riemannian geometry, we put $\beta = 0$ and assume $\psi = 1$ for simplicity, from (3.18) the homothetic vector becomes

$$\zeta = t\,\partial t + (1-a)x\,\partial x + \{(1-b) - m[(1-a)x]\}y\partial y + \{(1-d) - m[(1-a)x]\}z\partial z \quad (3.20)$$

where m=0

$$\zeta = t\partial t + (1-a)x\partial x + (1-b)y\partial y + (1-d)z\partial z \qquad (3.21)$$

We can not compare the results with that obtained in G.R, because in this case the components ζ^0 tends to infinity when $\beta = 0$

IV) The results by using Maple 17:

In maple (With differential geometry): with tensor: we get the Christoffel symbol of first kind, Christoffel symbol of second kind, Killing vector and Killing equations hence the ten homothetic equations in Rumanian geometry where $(F1, F2, F3, F4) \equiv (\zeta^0, \zeta^1, \zeta^2, \zeta^3)$, and program solve the ten equations as a system which gives many sets of solutions see Appendix(1), as:

$$A(t) = -\frac{C_1}{\psi}; B(t) = C(t) = \frac{\sqrt{C_1(-2mt\psi + C_1 * C_4)}}{C_1}$$

$$F1 = C_1 x + \psi t + C_2$$

$$F2 = \psi x + \frac{\psi^2 t}{C_1} + C3$$

$$F3 = -C_3 my + \psi y\left(\frac{C_2 m}{C_1} + 1\right) + C_7 z + C_8$$

$$F3 = -C_7 y + m\psi z\left(\frac{C_2}{C_1} - 1\right) + C_9$$

Where i is a positive integer Ci are arbitrary constant .

We use maple 17 to solve the homothetic equation in Lyra geometry see Appendix(2)

By choose a suitable simultaneous PDF Which give many solution as :

$$A(t) = \frac{1}{\sqrt{F_7(t)}}, B(t) = -\frac{1}{\sqrt{e^{2mx}F_{10}(t)}}, C(t) = -\frac{1}{\sqrt{e^{2mx}F_{13}(t)}}$$

$$F1 = \{\int \psi e^{\int \frac{\beta}{2}dt} dt + F_2(x,y,z)\} e^{\int -\frac{\beta}{2}dt}$$

$$F2 = F_6(x,y,z) + \int F_7(t) F1_{,x} dt$$

$$F3 = F_9(x,y,z) + \int F_{10}(t) F1_{,y} dt$$

$$F4 = F_{12}(x,y,z) + \int F_{13}(t) F1_{,z} dt$$

F_i are arbitrary functions

V) DISCUSSION

In this paper we get the ten homothetic equations for Bianchi type-V in Riemann geometry and in Lyra geometry and solve these ten equations by science of partial differential equation and also by Maple17 program and get the homothetic vector in the Riemann geometry and in Lyra geometry, from equation (3.1) and (3.11) ζ^0 is a function of t, from equation (3..13) ζ^1 is a function of x, from equation (3.16) ζ^2 is a function of x,y and from equation (3.17) ζ^3 is a function of x, z which is different in the case of Bianchi type-I, when m=0 in the equation (3.20) the homothetic vector field be as of Bianchi type-I. [17]

VI) CONCLUSION

The component homothetic vector field in Bianchi Type –V Model in Lyra Geometry has more variables than homothetic vector field in Bianchi Type –I and it will be the same like Bianchi Type –I if the constant m equal zero..

VII) RECOMMENDATIONS

It can be study the same equations for Bianchi type-V in teleparallel geometry and using maple to get the homothetic equations and its solution for more difficult models

Appendix(1)

> *with*(*DifferentialGeometry*) : *with*(*Tensor*) : ***Remian Geometry***

Remian Geometry

> *DGsetup*([*t*, x, y, *z*], *M*)

frame name: M

M > $g := evalDG(\, dt\,\&t\,dt - A^2(t)\,dx\,\&t\,dx - e^{2m\cdot x}B^2(t)\,dy\,\&t\,dy - e^{2m\cdot x}C^2(t)\,dz\&t\,dz)$

$$g := dt\,dt - A(t)^2\,dx\,dx - e^{2mx}\,B(t)^2\,dy\,dy - e^{2mx}\,C(t)^2\,dz\,dz$$

> $H := InverseMetric(g)$

$$H := D\_t\,D\_t - \frac{1}{A(t)^2}\,D\_x\,D\_x - \frac{e^{-2mx}}{B(t)^2}\,D\_y\,D\_y - \frac{e^{-2mx}}{C(t)^2}\,D\_z\,D\_z$$

M > $C1 := Christoffel(g, \text{"FirstKind"})$

$$\begin{aligned} C1 := {} & A(t)\,\dot{A}(t)\,dt\,dx\,dx + e^{2mx}\,B(t)\,\dot{B}(t)\,dt\,dy\,dy + e^{2mx}\,C(t)\,\dot{C}(t)\,dt\,dz\,dz - A(t)\,\dot{A}(t)\,dx\,dt\,dx \\ & - A(t)\,\dot{A}(t)\,dx\,dx\,dt + m\,e^{2mx}\,B(t)^2\,dx\,dy\,dy + m\,e^{2mx}\,C(t)^2\,dx\,dz\,dz - e^{2mx}\,B(t) \\ & \dot{B}(t)\,dy\,dt\,dy - m\,e^{2mx}\,B(t)^2\,dy\,dx\,dy - e^{2mx}\,B(t)\,\dot{B}(t)\,dy\,dy\,dt - m\,e^{2mx}\,B(t)^2\,dy\,dy\,dx \\ & - e^{2mx}\,C(t)\,\dot{C}(t)\,dz\,dt\,dz - m\,e^{2mx}\,C(t)^2\,dz\,dx\,dz - e^{2mx}\,C(t)\,\dot{C}(t)\,dz\,dz\,dt \\ & - m\,e^{2mx}\,C(t)^2\,dz\,dz\,dx \end{aligned}$$

M > $C2 := Christoffel(g, \text{"SecondKind"})$

$$C2 := A(t)\,\dot{A}(t)\,D\_t\,dx\,dx + e^{2mx}B(t)\,\dot{B}(t)\,D\_t\,dy\,dy + e^{2mx}C(t)\,\dot{C}(t)\,D\_t\,dz\,dz + \frac{\dot{A}(t)}{A(t)}\,D\_x\,dt\,dx + \frac{\dot{A}(t)}{A(t)}\,D\_x\,dx\,dt - \frac{m\,e^{2mx}B(t)^2}{A(t)^2}\,D\_x\,dy\,dy - \frac{m\,e^{2mx}C(t)^2}{A(t)^2}\,D\_x\,dz\,dz + \frac{\dot{B}(t)}{B(t)}\,D\_y\,dt\,dy + m\,D\_y\,dx\,dy + \frac{\dot{B}(t)}{B(t)}\,D\_y\,dy\,dt + m\,D\_y\,dy\,dx + \frac{\dot{C}(t)}{C(t)}\,D\_z\,dt\,dz + m\,D\_z\,dx\,dz + \frac{\dot{C}(t)}{C(t)}\,D\_z\,dz\,dt + m\,D\_z\,dz\,dx$$

M > K1 := KillingVectors(g);

$$K1 := \left[\frac{1}{m}D\_x - y\,D\_y - z\,D\_z,\ -D\_z,\ -D\_y\right]$$

M > LD := LieDerivative(K1, g)

$$LD := [0\,dt\,dt,\ 0\,dt\,dt,\ 0\,dt\,dt]$$

M > pde := KillingVectors(g, output = "pde");

$$pde := \Big\{0,\ \frac{1}{A(t)^2}\left(-e^{2mx}B(t)\left(\frac{d}{dt}B(t)\right)\_F1(t,x,y,z)\,A(t)^2 + m\,e^{2mx}B(t)^2\,\_F2(t,x,y,z) + \left(\frac{\partial}{\partial y}\_F3(t,x,y,z)\right)A(t)^2\right),\ \frac{1}{A(t)^2}\left(-e^{2mx}C(t)\left(\frac{d}{dt}C(t)\right)\_F1(t,x,y,z)\,A(t)^2 + m\,e^{2mx}C(t)^2\,\_F2(t,x,y,z) + \left(\frac{\partial}{\partial z}\_F4(t,x,y,z)\right)A(t)^2\right),$$

$$\frac{1}{2}\,\frac{-2\left(\frac{d}{dt}C(t)\right)\_F4(t,x,y,z) + \left(\frac{\partial}{\partial t}\_F4(t,x,y,z)\right)C(t) + \left(\frac{\partial}{\partial z}\_F1(t,x,y,z)\right)C(t)}{C(t)},$$

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial t}\_F2(t,x,y,z)\right)A(t) + \left(\frac{\partial}{\partial x}\_F1(t,x,y,z)\right)A(t) - 2\left(\frac{d}{dt}A(t)\right)\_F2(t,x,y,z)}{A(t)},$$

$$\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial t}\_F3(t,x,y,z)\right)B(t) - 2\left(\frac{d}{dt}B(t)\right)\_F3(t,x,y,z) + \left(\frac{\partial}{\partial y}\_F1(t,x,y,z)\right)B(t)}{B(t)},$$

$$-A(t)\left(\frac{d}{dt}A(t)\right)\_F1(t,x,y,z) + \frac{\partial}{\partial x}\_F2(t,x,y,z),\ \frac{1}{2}\frac{\partial}{\partial y}\_F4(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial z}\_F3(t,x,y,z),\ -m\_F3(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial x}\_F3(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial y}\_F2(t,x,y,z),\ -m\_F4(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial x}\_F4(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial z}\_F2(t,x,y,z),\ \frac{\partial}{\partial t}\_F1(t,x,y,z)\Big\}$$

M > (*Remian geometry*)

M >

$$sys2 := \left[\frac{1}{A(t)^2}\left(-e^{2mx}B(t)\left(\frac{d}{dt}B(t)\right)\_F1(t,x,y,z)\,A(t)^2 + m\,e^{2mx}B(t)^2\_F2(t,x,y,z) + \left(\frac{\partial}{\partial y}\_F3(t,x,y,z)\right)A(t)^2\right) = -2\,\psi\,e^{2mx}B(t)^2, \frac{1}{A(t)^2}\left(-e^{2mx}C(t)\left(\frac{d}{dt}C(t)\right)\_F1(t,x,y,z)\,A(t)^2 + m\,e^{2mx}C(t)^2\_F2(t,x,y,z) + \left(\frac{\partial}{\partial z}\_F4(t,x,y,z)\right)A(t)^2\right) = -2\,\psi e^{2m\cdot x}C^2(t),\right.$$

$$\frac{1}{2}\,\frac{-2\left(\frac{d}{dt}C(t)\right)\_F4(t,x,y,z) + \left(\frac{\partial}{\partial t}\_F4(t,x,y,z)\right)C(t) + \left(\frac{\partial}{\partial z}\_F1(t,x,y,z)\right)C(t)}{C(t)} = 0, \frac{1}{2}\,\frac{\left(\frac{\partial}{\partial t}\_F2(t,x,y,z)\right)A(t) + \left(\frac{\partial}{\partial x}\_F1(t,x,y,z)\right)A(t) - 2\left(\frac{d}{dt}A(t)\right)\_F2(t,x,y,z)}{A(t)}$$

$$= 0, \frac{1}{2}\,\frac{\left(\frac{\partial}{\partial t}\_F3(t,x,y,z)\right)B(t) - 2\left(\frac{d}{dt}B(t)\right)\_F3(t,x,y,z) + \left(\frac{\partial}{\partial y}\_F1(t,x,y,z)\right)B(t)}{B(t)}$$

$$= 0, -A(t)\left(\frac{d}{dt}A(t)\right)\_F1(t,x,y,z) + \frac{\partial}{\partial x}\_F2(t,x,y,z) = -2\,\psi\,A^2(t), \frac{1}{2}\frac{\partial}{\partial y}\_F4(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial z}\_F3(t,x,y,z) = 0, -m\_F3(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial x}\_F3(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial y}\_F2(t,x,y,z)$$

$$\left. = 0, -m\_F4(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial x}\_F4(t,x,y,z) + \frac{1}{2}\frac{\partial}{\partial z}\_F2(t,x,y,z) = 0, \frac{\partial}{\partial t}\_F1(t,x,y,z) = 2\,\psi\right];$$

M > *pdsolve*(*sys2*)

$$A(t) = \frac{2\,m\_C2\,(2\,\psi\,t + \_C1)}{m + \sqrt{-8\_C2\,m\,\psi + m^2}}, B(t) = (2\,\psi\,t + \_C1)^{\frac{1}{4}\frac{m + \sqrt{-8\_C2\,m\,\psi + m^2}}{\psi\_C2}}\_C5, C(t) = (2\,\psi\,t + \_C1)^{\frac{1}{4}\frac{m + \sqrt{-8\_C2\,m\,\psi + m^2}}{\psi\_C2}}\_C6, \_F1(t,x,y,z) = 2\,\psi\,t + \_C1, \_F2(t,x,y,z) = \_C2\,(2\,\psi\,t + \_C1)^2, \_F3(t,x,y,z) = \_C3\left(t + \frac{1}{2}\frac{\_C1}{\psi}\right)^{\frac{1}{2}\frac{m + \sqrt{-8\_C2\,m\,\psi + m^2}}{\psi\_C2}} e^{2mx},$$

$$\_F4(t,x,y,z) = (2\,\psi\,t + \_C1)^{\frac{1}{2}\frac{m + \sqrt{-8\_C2\,m\,\psi + m^2}}{\psi\_C2}}\_C4\,e^{2mx}$$

$$A(t) = -\frac{1}{2}\,\frac{(2\,\psi\,t + \_C1)\,m}{\psi}, B(t) = \_C3\,(2\,\psi\,t + \_C1), C(t) = \_C4\,(2\,\psi\,t + \_C1), \_F1(t,x,y,z) = 2\,\psi\,t + \_C1, \_F2(t,x,y,z) = 0, \_F3(t,x,y,z) = 0, \_F4(t,x,y,z) = \_C2\,(2\,\psi\,t + \_C1)^2\,e^{2mx}$$

$$A(t) = -\frac{\_C1}{\psi}, B(t) = -\frac{\sqrt{\_C1\,(-2\,m\,\psi\,t + \_C1\,\_C4)}}{\_C1}, C(t) = \frac{\sqrt{\_C1\,(-2\,m\,\psi\,t + \_C1\,\_C4)}}{\_C1}, \_F1(t,x,y,z) = \_C1\,x + \psi\,t + \_C2, \_F2(t,x,y,z) = \psi\,x + \frac{\psi^2\,t}{\_C1} + \_C3, \_F3(t,x,y,z) = -\_C3\,m\,y + \frac{\_C2\,m\,\psi\,y}{\_C1} + \psi\,y + \_C7\,z + \_C8, \_F4(t,x,y,z) = -\_C7\,y - \_C3\,m\,z + \frac{\_C2\,m\,\psi\,z}{\_C1} + \psi\,z + \_C9$$

$$\left\{A(t) = \_C3\,(2\,\psi\,t + \_C1), B(t) = \_C4\left(2\,(2\,\psi\,t + \_C1)^{\frac{1}{2}\frac{m\,\_C2}{\_C3^2\,\psi}}\,\psi\,t + (2\,\psi\,t + \_C1)^{\frac{1}{2}\frac{m\,\_C2}{\_C3^2\,\psi}}\,\_C1\right), C(t) = \_C5\left(2\,(2\,\psi\,t + \_C1)^{\frac{1}{2}\frac{m\,\_C2}{\_C3^2\,\psi}}\,\psi\,t + (2\,\psi\,t + \_C1)^{\frac{1}{2}\frac{m\,\_C2}{\_C3^2\,\psi}}\,\_C1\right), \_F1(t,x,y,z) = 2\,\psi\,t + \_C1, \_F2(t,x,y,z) = \_C2\,(2\,\psi\,t + \_C1)^2, \_F3(t,x,y,z) = 0, \_F4(t,x,y,z) = 0\right\}, \left\{A(t) = -\frac{\sqrt{2}\,\sqrt{(\_C3 - 4\,\psi)\,m\,\_C2}\,(2\,\psi\,t + \_C1)}{\_C3 - 4\,\psi}, B(t) = \_C5\,(2\,\psi\,t + \_C1)^{\frac{1}{4}\frac{\_C3}{\psi}}, C(t) = \_C6\,(2\,\psi\,t + \_C1)^{\frac{1}{4}\frac{\_C3}{\psi}}, \_F1(t,x,y,z) = 2\,\psi\,t + \_C1, \_F2(t,x,y,z) = \_C2\,(2\,\psi\,t + \_C1)^2, \_F3(t,x,y,z) = (2\,\psi\,t + \_C1)^{\frac{1}{2}\frac{\_C3}{\psi}}\,\_C4\,e^{2\,m\,x}, \_F4(t,x,y,z) = 0\right\}$$

Appendix(2)

M > *nops*(*pde*);

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M > *nops*(*list*);

1

M > *with*(*PDEtools, casesplit, declare*);

[*casesplit, declare*]

M > *with*(*DEtools, gensys*);

[*gensys*]

M > *IN LYRA GEOMETRY*

M > *declare*((F1, F2, F3, F4)(*t*, x, y, *z*), β(*t*));

M >

$$sys1 := \left[A^2(t) \cdot \left(\frac{\partial}{\partial t} \_F2(t,x,y,z) \right) - \left(\frac{\partial}{\partial x} \_F1(t,x,y,z) \right) = 0, \left(\frac{\partial}{\partial y} \_F1(t,x,y,z) \right) - B^2(t) \cdot e^{2mx} \cdot \left(\frac{\partial}{\partial t} \_F3(t,x,y,z) \right) = 0, \left(\frac{\partial}{\partial z} \_F1(t,x,y,z) \right) - C^2(t) \cdot e^{2mx} \cdot \left(\frac{\partial}{\partial t} \_F4(t,x,y,z) \right) = 0, \left(\frac{1}{A(t)} \frac{d}{dt} A(t) + \frac{\beta(t)}{2} \right) \_F1(t,x,y,z) + \frac{\partial}{\partial x} \_F2(t,x,y,z) = \psi, \left(\frac{1}{B(t)} \frac{d}{dt} B(t) + \frac{\beta(t)}{2} \right) \_F1(t,x,y,z) + m\_F2(t,x,y,z) + \frac{\partial}{\partial y} \_F3(t,x,y,z) = \psi, \left(\frac{1}{C(t)} \frac{d}{dt} C(t) + \frac{\beta(t)}{2} \right) \_F1(t,x,y,z) + m\_F2(t,x,y,z) + \frac{\partial}{\partial z} \_F4(t,x,y,z) = \psi, B^2(t) \cdot e^{2mx} \cdot \left(\frac{\partial}{\partial x} \_F3(t,x,y,z) \right) + A^2(t) \frac{\partial}{\partial y} \_F2(t,x,y,z) = 0, C^2(t) \cdot e^{2mx} \frac{\partial}{\partial x} \_F4(t,x,y,z) + A^2(t) \frac{\partial}{\partial z} \_F2(t,x,y,z) = 0, B^2(t) \frac{\partial}{\partial y} \_F4(t,x,y,z) + C^2(t) \frac{\partial}{\partial z} \_F3(t,x,y,z) = 0, \frac{\partial}{\partial t} \_F1(t,x,y,z) + \frac{\beta(t)}{2} \_F1(t,x,y,z) = \psi \right];$$

$$sys1 := \left[A(t)^2 \_F2_t - \_F1_x = 0, \_F1_y - B(t)^2 e^{2mx} \_F3_t = 0, \_F1_z - C(t)^2 e^{2mx} \_F4_t = 0, \left(\frac{A_t}{A(t)} + \frac{1}{2}\beta \right) \_F1(t,x,y,z) + \_F2_x = \psi, \left(\frac{B_t}{B(t)} + \frac{1}{2}\beta \right) \_F1(t,x,y,z) + m\_F2(t,x,y,z) + \_F3_y = \psi, \left(\frac{C_t}{C(t)} + \frac{1}{2}\beta \right) \_F1(t,x,y,z) + m\_F2(t,x,y,z) + \_F4_z = \psi, B(t)^2 e^{2mx} \_F3_x + A(t)^2 \_F2_y = 0, C(t)^2 e^{2mx} \_F4_x + A(t)^2 \_F2_z = 0, B(t)^2 \_F4_y + C(t)^2 \_F3_z = 0, \_F1_t + \frac{1}{2}\beta \_F1(t,x,y,z) = \psi \right]$$

M > *nops*(*sys1*);

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M > for _*eq* in *sys1* do _*eq* end do;

$$A(t)^2 \_F2_t - \_F1_x = 0$$

$$\_F1_y - B(t)^2 e^{2mx} \_F3_t = 0$$

$$\_F1_z - C(t)^2 e^{2mx} \_F4_t = 0$$

$$\left(\frac{A_t}{A(t)} + \frac{1}{2}\beta \right) \_F1(t,x,y,z) + \_F2_x = \psi$$

$$\left(\frac{B_t}{B(t)} + \frac{1}{2}\beta \right) \_F1(t,x,y,z) + m\_F2(t,x,y,z) + \_F3_y = \psi$$

$$\left(\frac{C_t}{C(t)} + \frac{1}{2}\beta \right) \_F1(t,x,y,z) + m\_F2(t,x,y,z) + \_F4_z = \psi$$

$$B(t)^2 e^{2mx} \_F3_x + A(t)^2 \_F2_y = 0$$

$$C(t)^2 e^{2mx} \_F4_x + A(t)^2 \_F2_z = 0$$

$$B(t)^2 \_F4_y + C(t)^2 \_F3_z = 0$$

$$\_F1_t + \frac{1}{2}\beta \_F1(t,x,y,z) = \psi$$

M > *pdsolve*(*sys1*)

[Length of output exceeds limit of 1000000]

M > pde1 := $A(t)^2 \frac{\partial}{\partial t}\_F2(t,x,y,z) - \frac{\partial}{\partial x}\_F1(t,x,y,z) = 0;$

$pde1 := A(t)^2 \_F2_t - \_F1_x = 0$

M > $pde2 := \frac{\partial}{\partial y}\_F1(t,x,y,z) - B(t)^2 e^{2mx} \frac{\partial}{\partial t}\_F3(t,x,y,z) = 0;$

$pde2 := \_F1_y - B(t)^2 e^{2mx} \_F3_t = 0$

M > $pde3 := \frac{\partial}{\partial z}\_F1(t,x,y,z) - C(t)^2 e^{2mx} \frac{\partial}{\partial t}\_F4(t,x,y,z) = 0;$

$pde3 := \_F1_z - C(t)^2 e^{2mx} \_F4_t = 0$

M > $pde4 := \left(\frac{\dot{A}(t)}{A(t)} + \frac{1}{2}\beta(t)\right)\_F1(t,x,y,z) + \frac{\partial}{\partial x}\_F2(t,x,y,z) = \psi;$

$pde4 := \left(\frac{A_t}{A(t)} + \frac{1}{2}\beta\right)\_F1(t,x,y,z) + \_F2_x = \psi$

M > $pde5 := \left(\frac{\dot{B}(t)}{B(t)} + \frac{1}{2}\beta(t)\right)\_F1(t,x,y,z) + m\_F2(t,x,y,z) + \frac{\partial}{\partial y}\_F3(t,x,y,z) = \psi;$

$pde5 := \left(\frac{B_t}{B(t)} + \frac{1}{2}\beta\right)\_F1(t,x,y,z) + m\_F2(t,x,y,z) + \_F3_y = \psi$

M > $pde6 := \left(\frac{\dot{C}(t)}{C(t)} + \frac{1}{2}\beta(t)\right)\_F1(t,x,y,z) + m\_F2(t,x,y,z) + \frac{\partial}{\partial z}\_F4(t,x,y,z) = \psi;$

$pde6 := \left(\frac{C_t}{C(t)} + \frac{1}{2}\beta\right)\_F1(t,x,y,z) + m\_F2(t,x,y,z) + \_F4_z = \psi$

M > $pde7 := B(t)^2 e^{2mx} \frac{\partial}{\partial x}\_F3(t,x,y,z) + A(t)^2 \frac{\partial}{\partial y}\_F2(t,x,y,z) = 0;$

$pde7 := B(t)^2 e^{2mx} \_F3_x + A(t)^2 \_F2_y = 0$

M > $pde8 := C(t)^2 e^{2mx} \frac{\partial}{\partial x}\_F4(t,x,y,z) + A(t)^2 \frac{\partial}{\partial z}\_F2(t,x,y,z) = 0;$

$pde8 := C(t)^2 e^{2mx} \_F4_x + A(t)^2 \_F2_z = 0$

M > $pde9 := B(t)^2 \frac{\partial}{\partial y}\_F4(t,x,y,z) + C(t)^2 \frac{\partial}{\partial z}\_F3(t,x,y,z) = 0;$

$pde9 := B(t)^2 \_F4_y + C(t)^2 \_F3_z = 0$

M > pde10 := $\frac{\partial}{\partial t}\_F1(t,x,y,z) + \frac{1}{2}\beta(t)\_F1(t,x,y,z) = \psi;$

$pde10 := \_F1_t + \frac{1}{2}\beta\_F1(t,x,y,z) = \psi$

M > *pdsolve*([pde10]);

$$\left\{\_F1(t,x,y,z) = \left(\int \psi\, e^{\frac{1}{2}\int \beta\, dt} dt + \_F2(x,y,z)\right) e^{\int\left(-\frac{1}{2}\beta\right) dt}\right\}$$

M > *pdsolve*([*pde4*, *pde5*, *pde6*, *pde10*])

$$\Big\{A(t)=A(t), B(t)=B(t), C(t)=C(t), \_F1(t,x,y,z)=2\_C1+\_C2+\frac{t}{\_c_1}, \_F2(t,x,y,z)= -\frac{\left(2A_t\_C1\_c_1+A_t\_C2\_c_1+A_t t-A(t)\right)x}{A(t)\_c_1}+\_F5_y, \_F3(t,x,y,z)=-\_F5(t,y,z)\,m+\left(\frac{2mxA_t\_C1}{A(t)}+\frac{mxA_t\_C2}{A(t)}+\frac{mxA_t t}{\_c_1 A(t)}-\frac{mx}{\_c_1}-\frac{2B_t\_C1}{B(t)}-\frac{B_t\_C2}{B(t)}-\frac{B_t t}{\_c_1 B(t)}+\frac{1}{\_c_1}\right)y+\_F6(t,x,z), \_F4(t,x,y,z)=\int\left(-\_F5_y m\right)dz+\left(\frac{2mxA_t\_C1}{A(t)}+\frac{mxA_t\_C2}{A(t)}+\frac{mxA_t t}{\_c_1A(t)}-\frac{2C_t\_C1}{C(t)}-\frac{C_t\_C2}{C(t)}-\frac{mx}{\_c_1}-\frac{C_t t}{\_c_1C(t)}+\frac{1}{\_c_1}\right)z+\_F7(t,x,y), \beta=\frac{2\psi\_c_1-2}{(2\_C1+\_C2)\_c_1+t}\Big\}$$

M > *pdsolve*([*pde7, pde8, pde9, pde10*])

$$A(t)=A(t), B(t)=0, C(t)=-\frac{\sqrt{-e^{2mx}\_F10(t)}\,A(t)}{e^{2mx}}, \_F1(t,x,y,z)=\_C2+2\_C1+\frac{t}{\_c_1}, \_F2(t,x,y,z)=\_F8(t,x)+\left(\int\frac{\partial}{\partial x}\_F7(t,x,z)\,dz\right)\_F10(t), \_F3(t,x,y,z)=\_F5(t,x,y), \_F4(t,x,y,z)=\_F7(t,x,z)+\_F6(t,y,z), \beta(t)=\frac{2\psi\_c_1-2}{(2\_C1+\_C2)\_c_1+t}$$

$$\Big\{A(t)=A(t), B(t)=\frac{\sqrt{-e^{2mx}\_F9(t)}\,A(t)}{e^{2mx}}, C(t)=-\frac{\left(-\_F9(t)^2\right)^{1/4}A(t)}{e^{mx}}, \_F1(t,x,y,z)=\_C2+2\_C1+\frac{t}{\_c_1}, \_F2(t,x,y,z)=\_F7(t,x)+\left(\int\frac{\partial}{\partial x}\_F5(t,x,y)\,dy\right)\_F9(t), \_F3(t,x,y,z)=\_F5(t,x,y), \_F4(t,x,y,z)=\_F6(t,z), \beta(t)=\frac{2\psi\_c_1-2}{(2\_C1+\_C2)\_c_1+t}\Big\}$$

$$\Big\{A(t)=A(t), B(t)=-\frac{\sqrt{e^{2mx}\_F8(t)\_F11(t)}\,A(t)}{e^{2mx}\_F8(t)}, C(t)=\frac{\sqrt{-e^{2mx}\_F11(t)}\,A(t)}{e^{2mx}}, \_F1(t,x,y,z)=\_C2+2\_C1+\frac{t}{\_c_1}, \_F2(t,x,y,z)=\_F9(t,x)+\left(\int\frac{\partial}{\partial x}\_F6(t,x,z)\,dz\right)\_F11(t), \_F3(t,x,y,z)=\_F5(t,y,z), \_F4(t,x,y,z)=\_F6(t,x,z)+\left(\int\frac{\partial}{\partial z}\_F5(t,y,z)\,dy\right)\_F8(t), \beta(t)=\frac{2\psi\_c_1-2}{(2\_C1+\_C2)\_c_1+t}\Big\}$$

M > *pdsolve*([*pde1, pde2, pde3*])

$$A(t)=0,\ B(t)=\frac{1}{\sqrt{e^{2\,mx}\,\_F10(t)}},\ C(t)=0,\ \_F1(t,x,y,z)=\_F5(t,y),\ \_F2(t,x,y,z)=\_F2(t,x,y,z),\ \_F3(t,x,y,z)=\_F11(y)+\int \_F10(t)\,\frac{\partial}{\partial y}\,\_F5(t,y)\,\mathrm{d}t+\_F8(y,z)+\_F6(x,y,z),\ \_F4(t,x,y,z)=\_F4(t,x,y,z)$$

$$A(t)=-\frac{1}{\sqrt{\_F10(t)}},\ B(t)=\frac{1}{\sqrt{e^{2\,mx}\,\_F15(t)}},\ C(t)=C(t),\ \_F1(t,x,y,z)=\_F5(t,x,y),\ \_F2(t,x,y,z)=\_F9(y,x)+\int \_F10(t)\,\frac{\partial}{\partial x}\,\_F5(t,x,y)\,\mathrm{d}t+\_F6(x,y,z),\ \_F3(t,x,y,z)=\_F14(y,x)+\int \_F15(t)\,\frac{\partial}{\partial y}\,\_F5(t,x,y)\,\mathrm{d}t+\_F11(x,y,z),\ \_F4(t,x,y,z)=\_F16(x,y,z)$$

$$A(t)=-\frac{1}{\sqrt{\_F7(t)}},\ B(t)=-\frac{1}{\sqrt{e^{2\,mx}\,\_F10(t)}},\ C(t)=-\frac{1}{\sqrt{e^{2\,mx}\,\_F13(t)}},\ \_F1(t,x,y,z)=\_F1(t,x,y,z),\ \_F2(t,x,y,z)=\_F6(z,y,x)+\int \_F7(t)\,\frac{\partial}{\partial x}\,\_F1(t,x,y,z)\,\mathrm{d}t,\ \_F3(t,x,y,z)=\_F9(z,y,x)+\int \_F10(t)\,\frac{\partial}{\partial y}\,\_F1(t,x,y,z)\,\mathrm{d}t,\ \_F4(t,x,y,z)=\_F12(z,y,x)+\int \_F13(t)\,\frac{\partial}{\partial z}\,\_F1(t,x,y,z)\,\mathrm{d}t$$

$$A(t)=-\frac{1}{\sqrt{\_F7(t)}},\ B(t)=\frac{1}{\sqrt{e^{2\,mx}\,\_F10(t)}},\ C(t)=\frac{1}{\sqrt{e^{2\,mx}\,\_F13(t)}},\ \_F1(t,x,y,z)=\_F1(t,x,y,z),\ \_F2(t,x,y,z)=\_F6(z,y,x)+\int \_F7(t)\,\frac{\partial}{\partial x}\,\_F1(t,x,y,z)\,\mathrm{d}t,\ \_F3(t,x,y,z)=\_F9(z,y,x)+\int \_F10(t)\,\frac{\partial}{\partial y}\,\_F1(t,x,y,z)\,\mathrm{d}t,\ \_F4(t,x,y,z)=\_F12(z,y,x)+\int \_F13(t)\,\frac{\partial}{\partial z}\,\_F1(t,x,y,z)\,\mathrm{d}t$$

M > *pdsolve*([*pde7*, *pde8*, *pde9*, *pde10*])

$$A(t)=A(t),\ B(t)=-\frac{\sqrt{-e^{2\,mx}\,\_F9(t)}\,A(t)}{e^{2\,mx}},\ C(t)=\frac{\mathrm{I}\left(-\_F9(t)^2\right)^{1/4}A(t)}{e^{mx}},\ \_F1(t,x,y,z)=\_C2+2\,\_C1+\frac{t}{\_c_1},\ \_F2(t,x,y,z)=\_F7(t,x)+\left(\int \frac{\partial}{\partial x}\,\_F5(t,x,y)\,\mathrm{d}y\right)\_F9(t),\ \_F3(t,x,y,z)=\_F5(t,x,y),\ \_F4(t,x,y,z)=\_F6(t,z),\ \beta(t)=\frac{2\,\psi\,\_c_1-2}{(2\,\_C1+\_C2)\,\_c_1+t}$$

$$A(t)=A(t),\ B(t)=\frac{\sqrt{-e^{2\,mx}\,\_F9(t)}\,A(t)}{e^{2\,mx}},\ C(t)=\frac{\sqrt{-e^{2\,mx}\,\_F12(t)}\,A(t)}{e^{2\,mx}},\ \_F1(t,x,y,z)=\_C2+2\,\_C1+\frac{t}{\_c_1},\ \_F2(t,x,y,z)=\_F10(t,x)+\left(\int \frac{\partial}{\partial x}\,\_F6(t,x,z)\,\mathrm{d}z\right)\_F12(t)+\left(\int \frac{\partial}{\partial x}\,\_F5(t,x,y)\,\mathrm{d}y\right)\_F9(t),\ \_F3(t,x,y,z)=\_F5(t,x,y),\ \_F4(t,x,y,z)=\_F6(t,x,z),\ \beta(t)=\frac{2\,\psi\,\_c_1-2}{(2\,\_C1+\_C2)\,\_c_1+t}$$

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الملخص: يتم استنتاج المعادلات العشرة الهمو ثيتيك لنموذج بيانكا 5 وذلك في هندسة ريمان وهندسة ليرا وكذلك حل المعادلات في الهندستين باستخدام طرق المعادلات التفاضلية الجزئية بفصل المتغيرات والمعادلات الخطية وكذلك باستخدام برنامج مابل 17 ومنها نستنتج متجه الهموثيتيك، نقارن بين المتجة في حالة بيانكا من النوع الخامس وبيانكامن النوع الاول.

الكلمات المفتاحية: بيانكا 5 ،كريستوفل سيمبول ، هندسة ريمان، هندسة ليرا، متجه الهموثتك